Quantum cavity optomechanics with nanomembranes

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Pisa CNR, June 10 2013

Outline of the talk

1. **Introduction** to cavity optomechanics: the example of a thin membrane within a Fabry-Perot cavity

2. **Ground state cooling and optomechanically induced transparency (OMIT) (also with our experimental setup in Camerino)**

3. **Proposal** for a **quantum optomechanical interface between microwave and optical signals**

INTRODUCTION

Micro- and nano-(opto)-electro-mechanical devices, i.e., MEMS, MOEMS and NEMS are extensively used for various technological applications :

• **high-sensitive sensors** (accelerometers, atomic force microscopes, mass sensors….)
• **actuators** (in printers, electronic devices…)

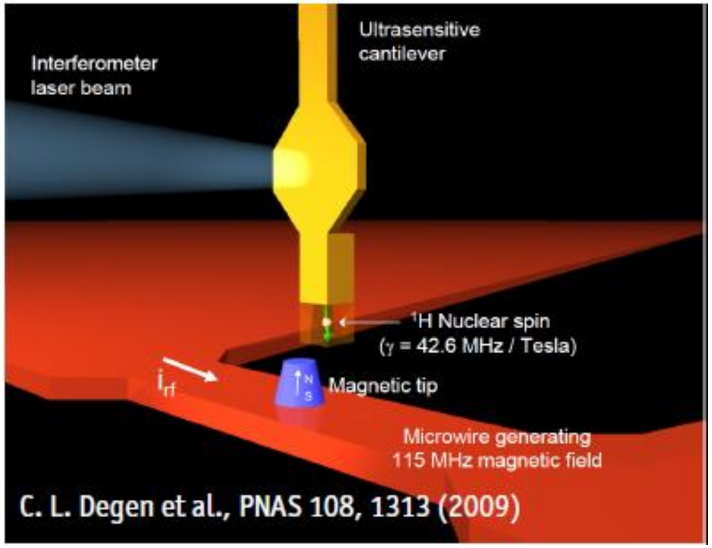
• These devices operate in the **classical regime** for both the electromagnetic field and the motional degree of freedom

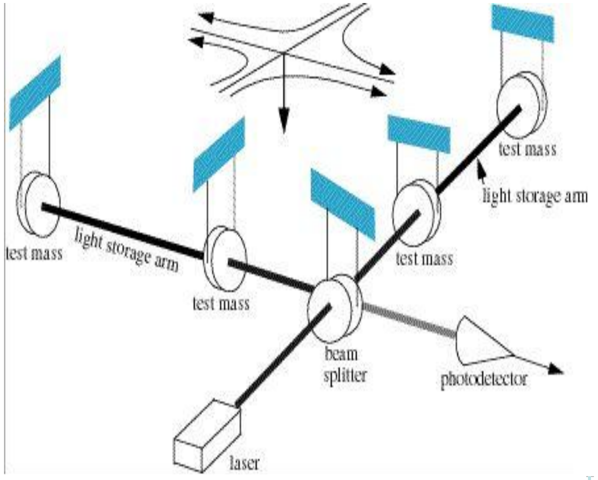
However very recently **cavity optomechanics** has emerged as a new field with **two elements of originality**:

1. the opportunities offered by **entering the quantum regime for these devices**
2. The crucial role played by an **optical (electromagnetic) cavity**

Why entering the quantum regime for opto- and electro-mechanical systems ?

1. **quantum-limited sensors**, i.e., working at the sensitivity limits imposed by Heisenberg uncertainty principle





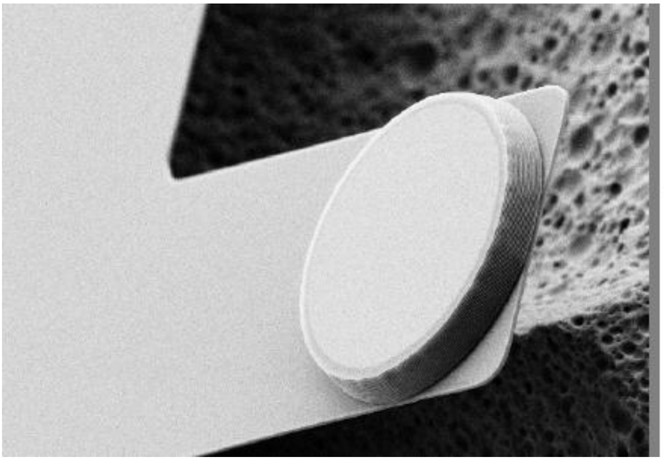
VIRGO (Pisa)

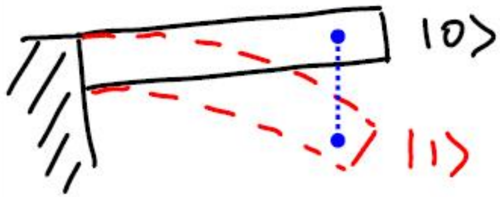
Nano-scale: **Single-spin MRFM**
D. Rugar group, IBM Almaden

Macro-scale: **gravitational wave interferometers** (VIRGO, LIGO)

Detection of extremely weak signal, forces and displacements

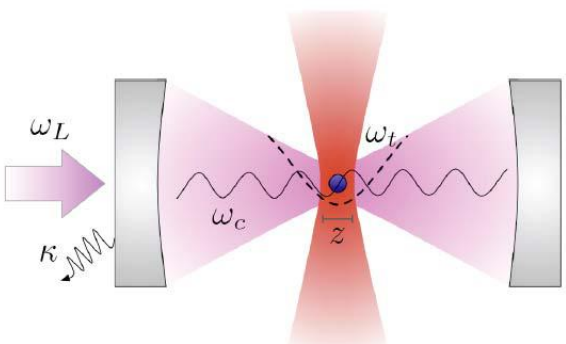
2. exploring the **boundary between the classical macroscopic world and the quantum microworld** (how far can we go in the demostration of macroscopic quantum phenomena ?)



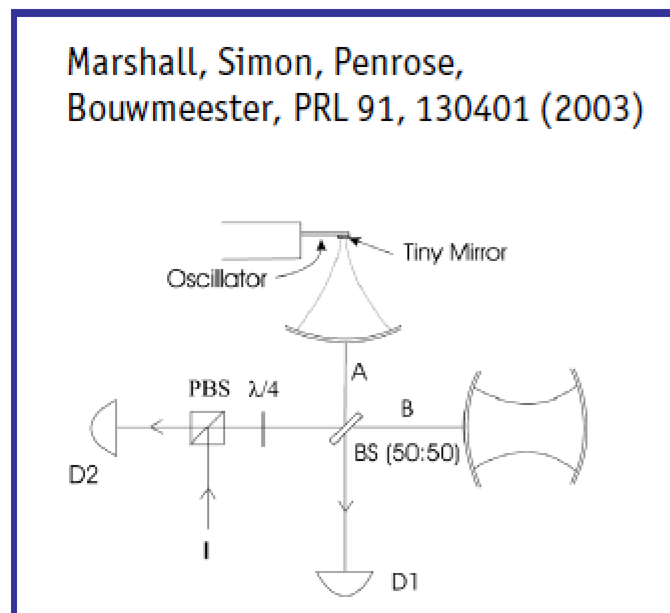


Superposition of macroscopically distinct states?

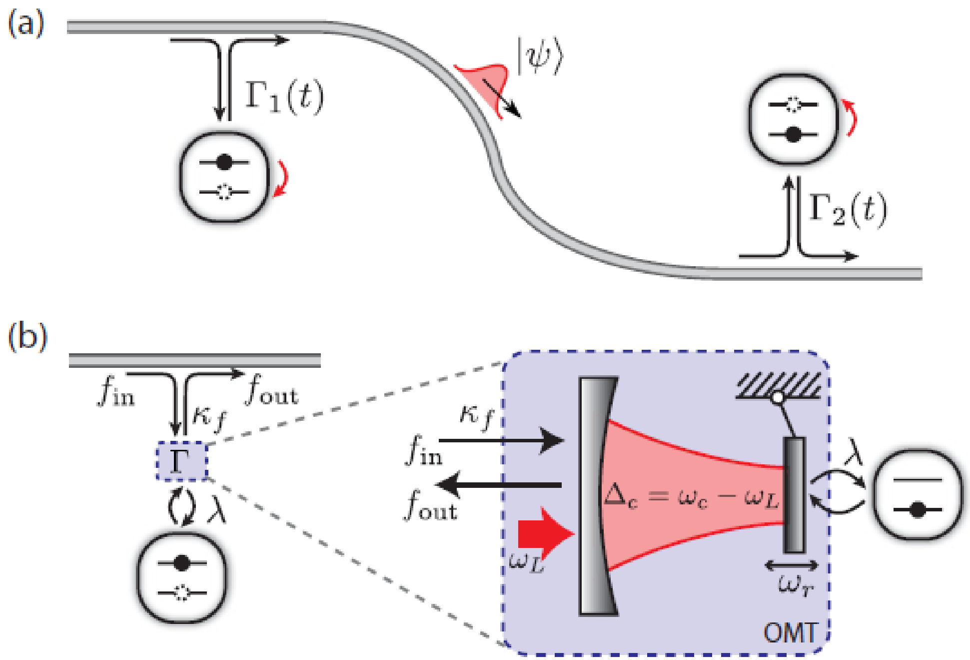
Optical detection of **Schrodinger cat states of a cantilever**



Cat states of optically levitated nanospheres (O. Romero-Isart, et al., Phys. Rev. Lett. 107, 020405 (2011).

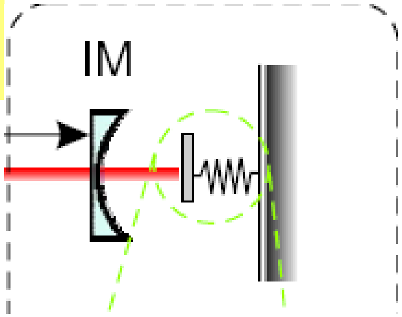


3. quantum information applications (optomechanical devices as **light-matter interfaces** and **transducers for quantum computing architectures, or long-distance quantum communication (optimal solid state qubits – optical photon transducer)**

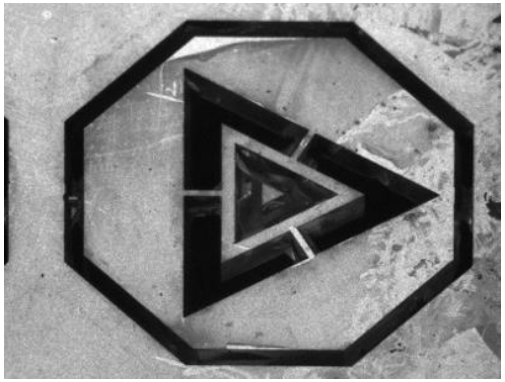


K. Stannigel, P. Rabl, A. S. Sørensen, P. Zoller, and M. D. Lukin, PRL 105, 220501 (2010)

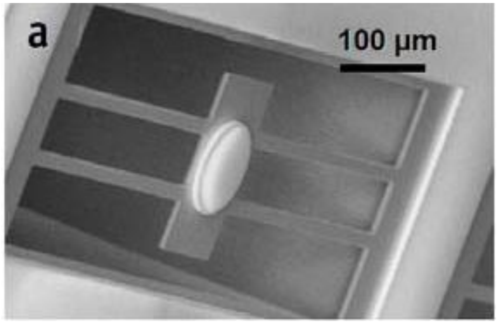
A large variety of cavity optomechanical devices recently developed



1. Fabry-Perot cavity with a moving micromirror

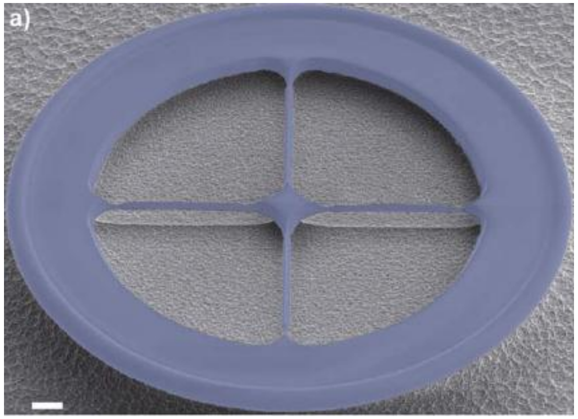


micropillar mirror (Heidmann, Paris)

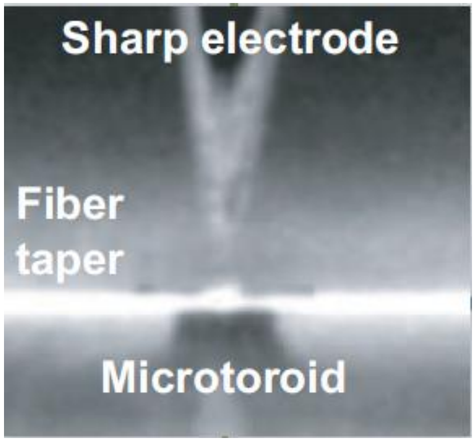


Monocrystalline Si cantilever, (Aspelmeyer, Vienna)

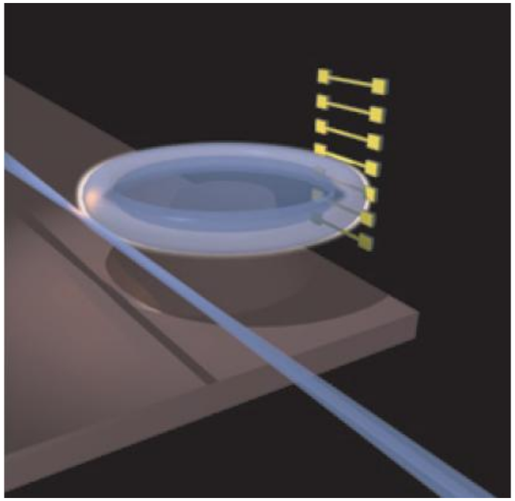
2. Silica toroidal optical microcavities



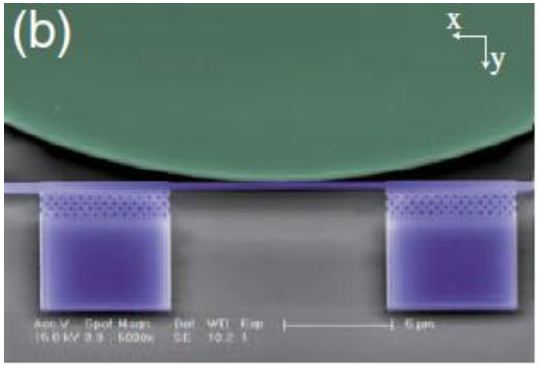
spoke-supported microresonator (Vahala-Caltech, Kippenberg, EPFL)



With electronic actuation, (Bowen, Brisbane)

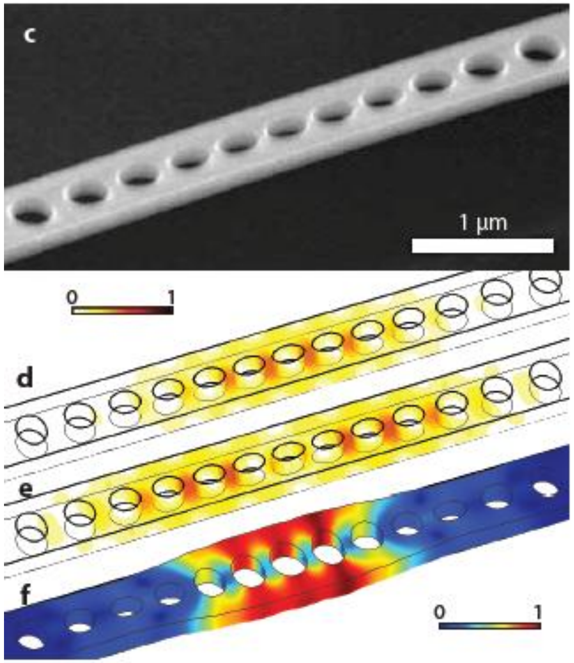


Evanescent coupling of a SiN nanowire to a toroidal microcavity (Kippenberg, EPFL)



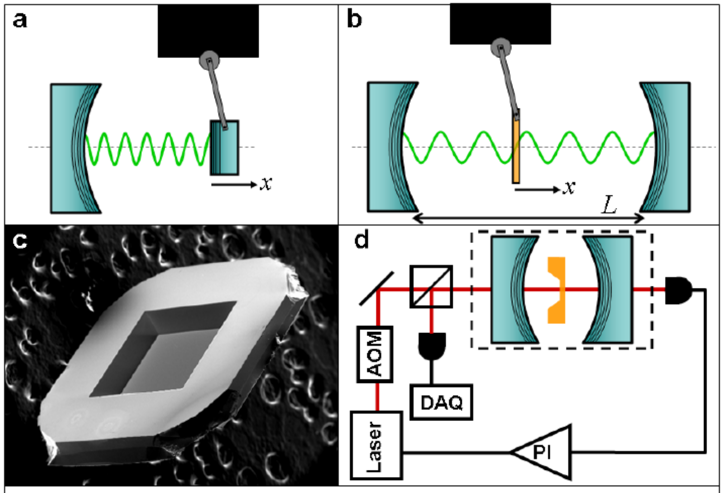
microdisk and a vibrating nanomechanical beam waveguide (Yale, H.Tang)

Radiation-pressure or dipole gradient coupling

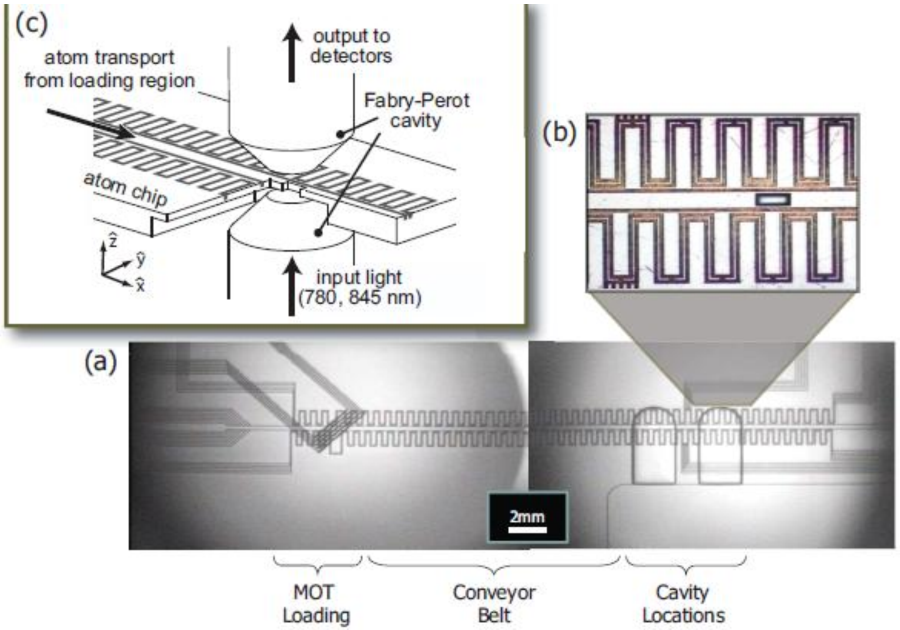


Silicon nanobeam photonic optomechanical cavity (Painter, Caltech)

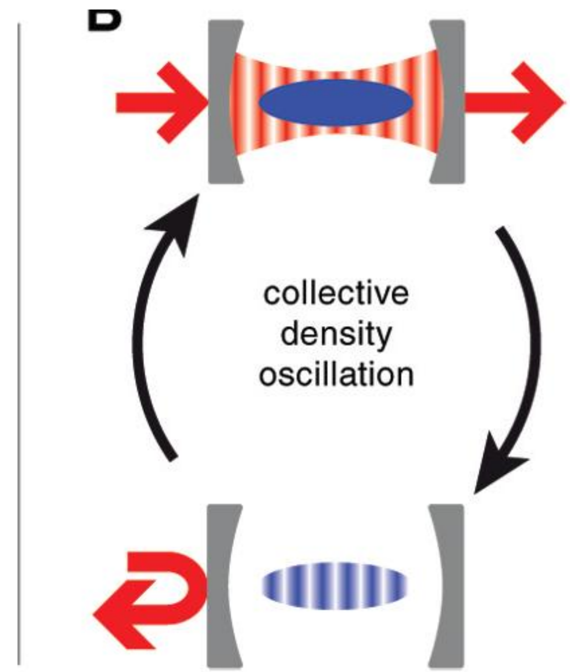
“membrane in the middle” scheme: Fabry-Perot cavity with a thin SiN membrane inside (J. Harris-Yale, Kimble-Caltech, Camerino, Regal-JILA)



ATOMIC CAVITY OPTOMECHANICS



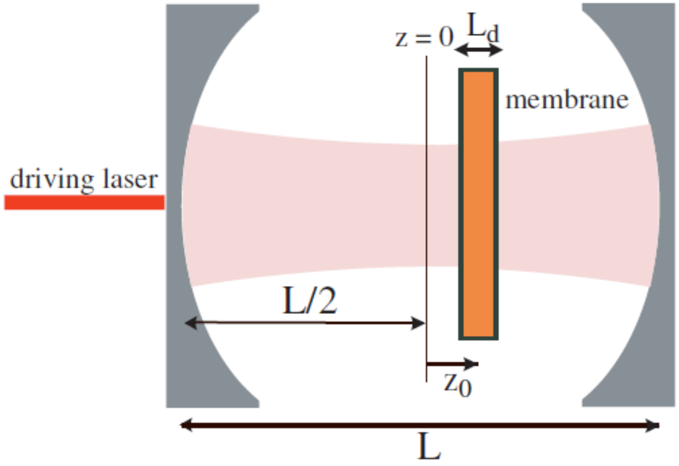
BEC mechanical oscillations within a Fabry-Perot cavity



Stamper-Kurn, Berkeley,
Esslinger, Zurich

The membrane-in-the-middle setup

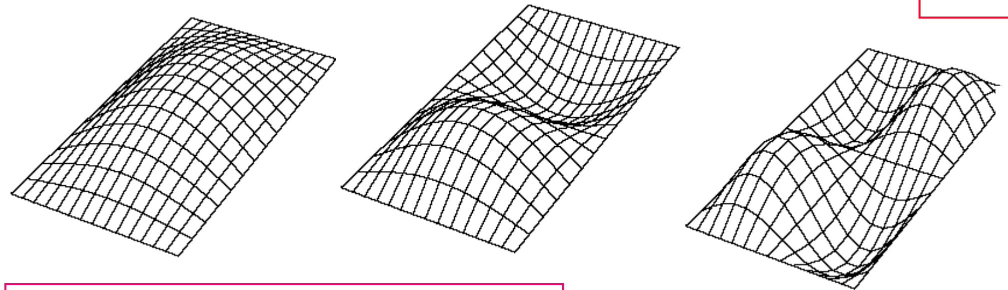
Many cavity modes (still Gaussian TEM_{mn} for an aligned membrane close to the waist)



$$H_{cav} = \sum_k \hbar\omega_k(z_0) a_k^+ a_k$$

Many vibrational modes $u_{mn}(x,y)$ of the membrane

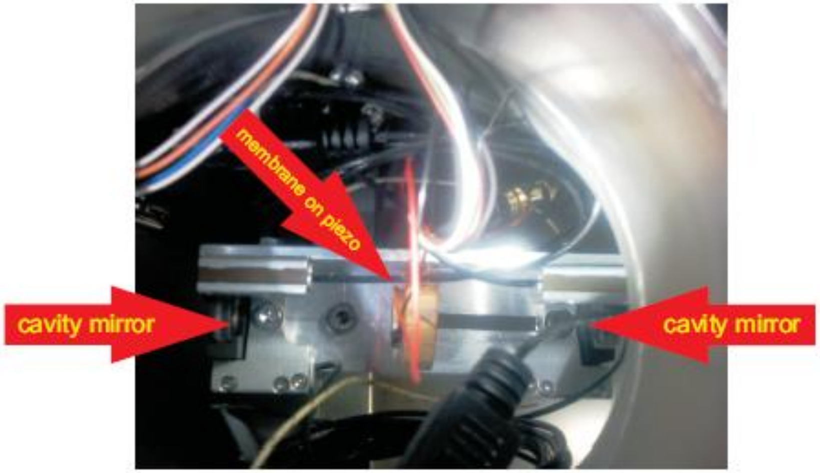
$$u_{mn}(x,y) = \sin\frac{n\pi x}{d}\sin\frac{m\pi y}{d}$$



$$\Omega_{nm} = \sqrt{\frac{\pi T}{\rho L_d d^2}\left(m^2+n^2\right)}$$

Vibrational frequencies

T = surface tension
ρ= SiN density,
L_d = membrane thickness
d = membrane side length
m,n = 1,2…



Optomechanical interaction due to radiation pressure

$$H_{\mathrm{int}} = -\int dxdy\, P_{rad}(x,y)\, z(x,y)$$

(at first order in z)

$$P_{rad}(x,y) = \varepsilon_0 \left(n_M^2 - 1\right) \int_{z_0 - L_d/2}^{z_0 + L_d/2} dz \left(\dot{\vec{E}}(x,y,z) \times \vec{B}(x,y,z)\right)_z$$

Radiation pressure field

$$z(x,y) = \sum_{n,m} \sqrt{\frac{\hbar}{M\Omega_{nm}}}\, q_{nm} u_{nm}(x,y)$$

Membrane axial deformation field

$$\hat{H}_{\mathrm{int}} = -\frac{2\hbar}{L} \sum_{l,k,n,m} \sqrt{\frac{\hbar \omega_l \omega_k}{M\Omega_{nm}}}\, \Theta_{nmlk} \Lambda_{lk} q_{nm} a_l^{+} a_k$$

C. Biancofiore et al., Phys. Rev. A **84,** 033814 (2011)

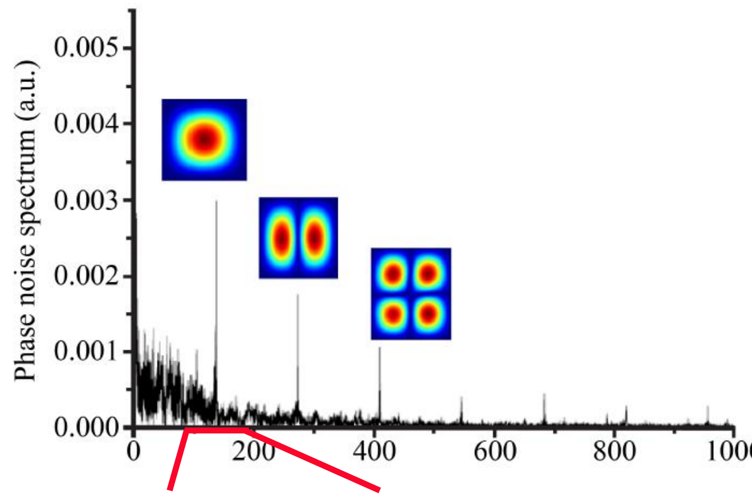
Nonlinear coupling describing photon scattering between cavity modes caused by the vibrating membrane

Simplified description: single mechanical oscillator, nonlinearly coupled to a single optical oscillator

Possible when:

• The external laser (with frequency $\omega_L \approx \omega_c$) **drives only a single cavity mode *a*** and scattering into the other cavity modes is negligible (no frequency close mode)

• A **bandpass filter** in the detection scheme can be used, isolating a single mechanical resonance



detection bandwidth

$$\hat{H} = \frac{\hbar\omega_m}{2}\left(p^2 + q^2\right) + \hbar\omega(q)a^+a + H_{drive}$$

Cavity optomechanics Hamiltonian

valid for a wide variety of systems

$$\hat{H}_{drive} = i\hbar\left(E e^{-i\omega_L t}a^+ - E^* e^{i\omega_L t}a\right)$$

$$E = \sqrt{\frac{2\kappa P_L}{\hbar\omega_L}}$$ amplitude of the driving laser with input power P_L

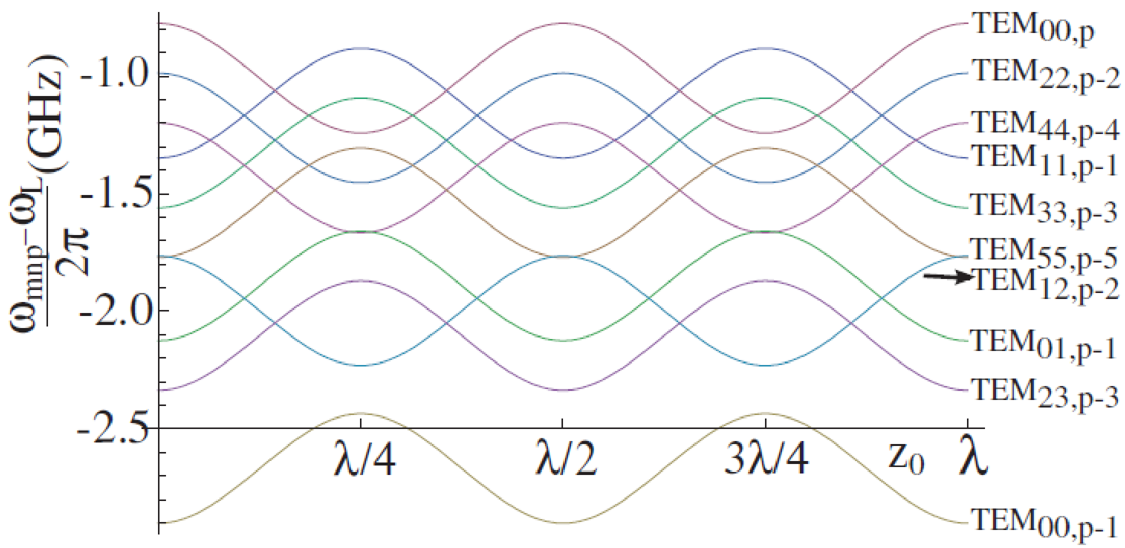
TUNABLE OPTOMECHANICAL INTERACTION by changing membrane position

Usual radiation pressure force interaction
$\Leftrightarrow$ first order expansion of $\omega(q)$

$$\boxed{\omega(q) = \omega_c - G_0 q}$$

Poor approximation at nodes and antinodes (where the dependence is quadratic)

$$\boxed{\omega(q) = \omega_0 + (-1)^p \arcsin\left\{\sqrt{R}\cos\left[2k_0 z_0(q)\right]\right\}}$$



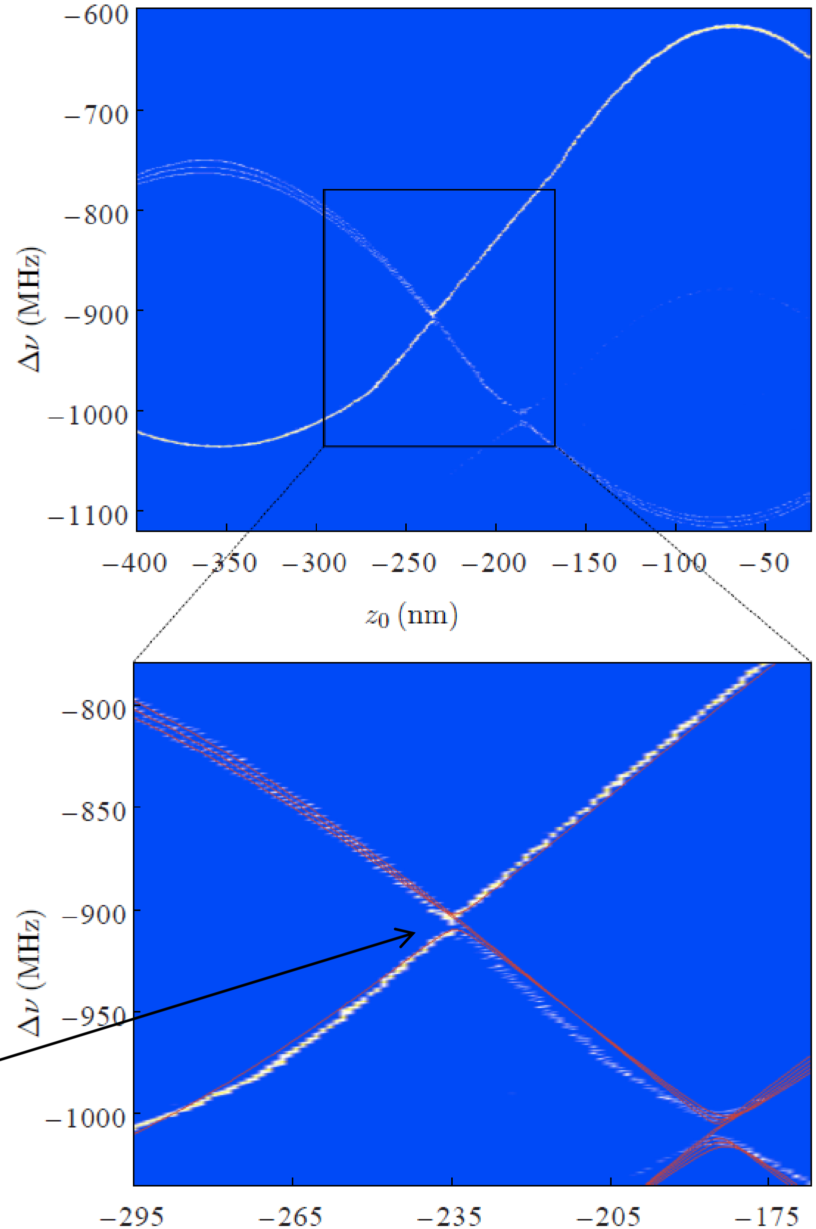
General periodic dependence for a **perfectly aligned membrane** with reflectivity R, **placed close to the waist**

Membrane misalignment (and shift from the waist) **couples the TEM$_{mn}$ cavity modes** via scattering

Splitting of degenerate modes and avoided crossings

linear combinations of nearby TEM$_{mn}$ modes become the new cavity modes: ***$\omega(q)$* is changed significantly: tunable optomechanical interaction**

Crossing between the TEM$_{00}$ singlet and the TEM$_{20}$ triplet



Experimental cavity frequencies with 0.21 mrad misalignment

(M. Karuza et al., J. Opt. 15 (2013) 025704)

At the membrane positions where there is an **avoided crossing, the optomechanical interaction becomes quadratic in the membrane position**

Avoided crossing between

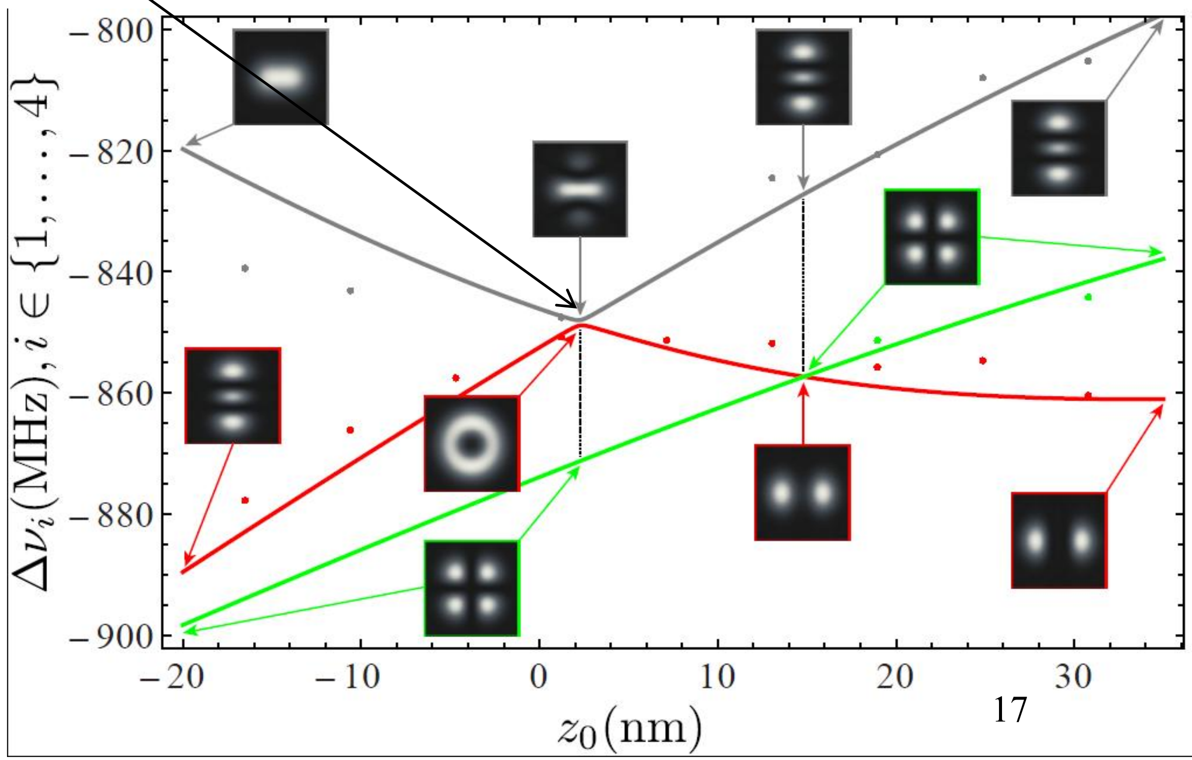
$$\left[|\mathrm{TEM}_{20,p-1}\rangle + |\mathrm{TEM}_{02,p-1}\rangle - |\mathrm{TEM}_{00,p}\rangle\right]/\sqrt{3}, \quad \text{and}$$
$$|\phi\rangle_{\mathrm{gray}} = \left[2|\mathrm{TEM}_{02,p-1}\rangle - |\mathrm{TEM}_{20,p-1}\rangle + |\mathrm{TEM}_{00,p}\rangle\right]/\sqrt{6}$$

$$\frac{\omega''(q)}{2\pi} = 4.46\,MHz/nm^2$$

Strong quadratic "dispersive" coupling

$$H_{\mathrm{int}} = \hbar\omega''(q)q^2 a^+ a$$

It allows the **nondestructive measurement** of mechanical energy: **possible detection of "phonon quantum jumps"**



Also damping and noise act on the system

- **The mechanical element** is in contact with **its environment at temperature *T***

Fluctuation-dissipation theorem $\Rightarrow$ presence of damping γ_m and of a **quantum Langevin force ξ** with correlation functions

$$\left\langle \xi(t)\xi(t')\right\rangle = \frac{\gamma_m}{\omega_m}\int \frac{d\omega}{2\pi} e^{i\omega(t-t')}\omega\left[\coth\left(\frac{\hbar\omega}{kT}\right)+1\right]$$

Gaussian quantum Brownian noise

- **The cavity mode** is damped by **photon leakage** with decay rate $\kappa \Rightarrow$ presence of a **vacuum input Langevin noise $a_{in}(t)$** with correlation functions

$$\left\langle a_{in}(t)a_{in}(t')\right\rangle = \left\langle a_{in}(t)^{+}a_{in}(t')\right\rangle = 0 \qquad \left\langle a_{in}(t)a_{in}(t')^{+}\right\rangle = \delta(t-t')$$

Gaussian vacuum noise

Vacuum electromagnetic field outside the cavity $\Leftrightarrow$ **reservoir at T ≈ 0 at optical frequencies**, because photon thermal occupancy is zero

$$\frac{\hbar\omega_a}{kT} >> 1 \qquad \Rightarrow \bar{N} = \left[e^{\frac{\hbar\omega_a}{kT}} - 1\right]^{-1} \approx 0$$

Including laser noise $\boxed{E \rightarrow \varepsilon(t) e^{-i\phi(t)}}$ ⇨

Heisenberg-Langevin equations in the rotating frame

$$\boxed{\begin{aligned} \dot{q} &= \omega_{\mathrm{m}} p, \\ \dot{p} &= -\omega_{\mathrm{m}} q - \gamma_{\mathrm{m}} p + G_0 a^{\dagger} a + \xi, \\ \dot{a} &= -\kappa a - \mathrm{i}(\Delta_0 - \dot{\phi} - G_0 q) a + \mathcal{E}(t) + \sqrt{2\kappa} \tilde{a}_{\mathrm{in}}(t) \end{aligned}}$$

Laser phase noise Laser amplitude noise

$$\Delta_0 = \omega_{\mathrm{c}} - \omega_0$$

$$G_0 = -\left(\frac{\mathrm{d}\omega_{\mathrm{c}}}{\mathrm{d}x}\right) \sqrt{\frac{\hbar}{m \omega_{\mathrm{m}}}} \quad \text{coupling}$$

- **Two quantum oscillators nonlinearly coupled and subject to noise and dissipation**
- strong **quantum correlations** between them better achieved with **strong driving**

Linearized Quantum Langevin equations for the quantum fluctuations

Strong laser driving and high-finesse cavity $\Rightarrow$ **steady-state with an intense intracavity field (amplitude α_s)** and deformed membrane.

We focus on the linearized dynamics of the **quantum fluctuations around this steady state** (only cavity mode is linearized $\Rightarrow$ **exact for $|\alpha_s| >> 1$**)

$$\boxed{\begin{aligned} \delta\dot{q} &= \omega_{\mathrm{m}}\delta p, \\ \delta\dot{p} &= -\omega_{\mathrm{m}}\delta q - \gamma_{\mathrm{m}}\delta p + G_0(\alpha_{\mathrm{s}}\delta a^{\dagger} + \alpha_{\mathrm{s}}^{*}\delta a) + \xi, \\ \delta\dot{a} &= -[\kappa + \mathrm{i}\Delta]\delta a + \mathrm{i}G_0\alpha_{\mathrm{s}}\delta q + \mathrm{i}\dot{\phi}\alpha_{\mathrm{s}} \\ &\quad +\varepsilon + \sqrt{2\kappa}\tilde{a}_{\mathrm{in}}. \end{aligned}}$$

$G = \sqrt{2}G_0|\alpha_s| >> G_0$

ENHANCED optomechanical coupling

$$\boxed{\Delta = \Delta_0 - G_0 q_{\mathrm{s}} = \Delta_0 - \frac{G_0^2|\alpha_{\mathrm{s}}|^2}{\omega_{\mathrm{m}}}}$$

Effective cavity detuning

EFFECT OF RADIATION PRESSURE ON THE MECHANICAL RESONATOR

1. Modified mechanical susceptibility

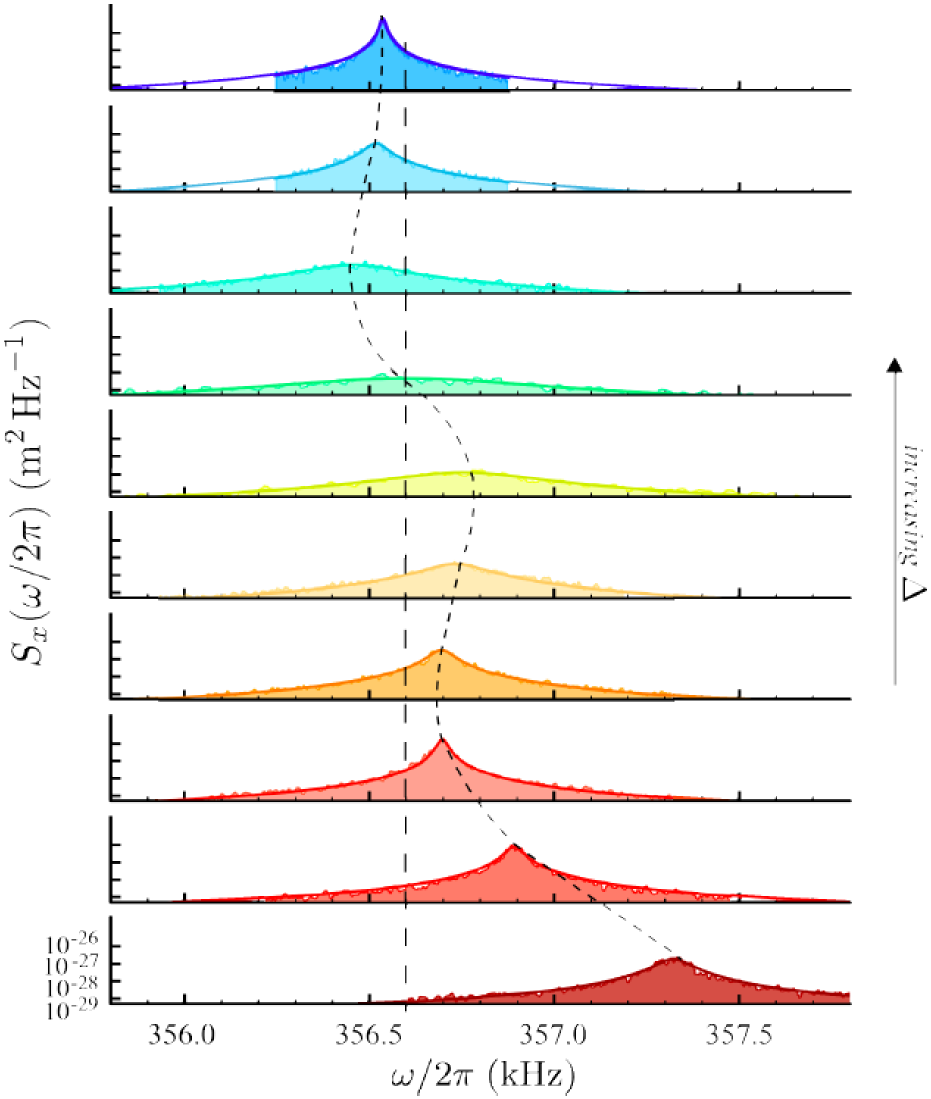
$$\boxed{\chi_{\mathrm{eff}}(\omega) = \frac{\Omega_{\mathrm{m}}}{\tilde{\Omega}_{\mathrm{m}}^2 - \omega^2 - \mathrm{i}\omega\gamma_{\mathrm{m}} - \frac{G^2\Delta\Omega_{\mathrm{m}}}{(\kappa_{\mathrm{T}} - \mathrm{i}\omega)^2 + \Delta^2}}}$$

$$\boxed{\gamma_m^{eff}(\omega) = \gamma_m + \frac{2G^2\Delta\omega_m\kappa}{|(\kappa - i\omega)^2 + \Delta^2|^2}}$$

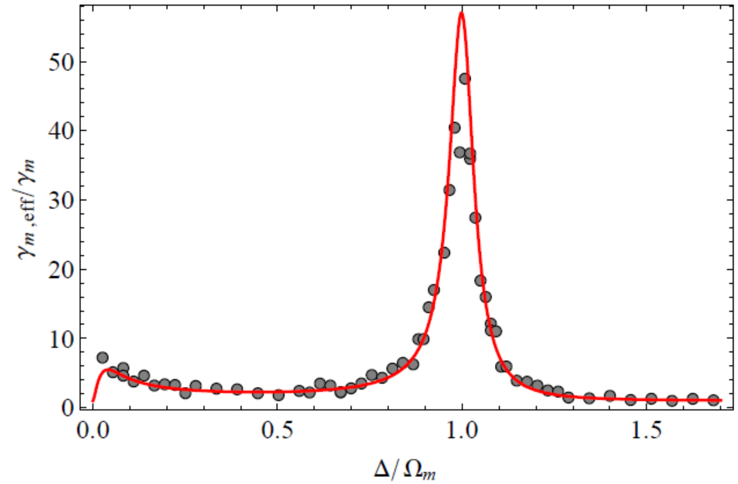
effective damping

- **shift of the mechanical resonance**
- **increased damping (**for $\Delta > 0$, red-detuned driving):

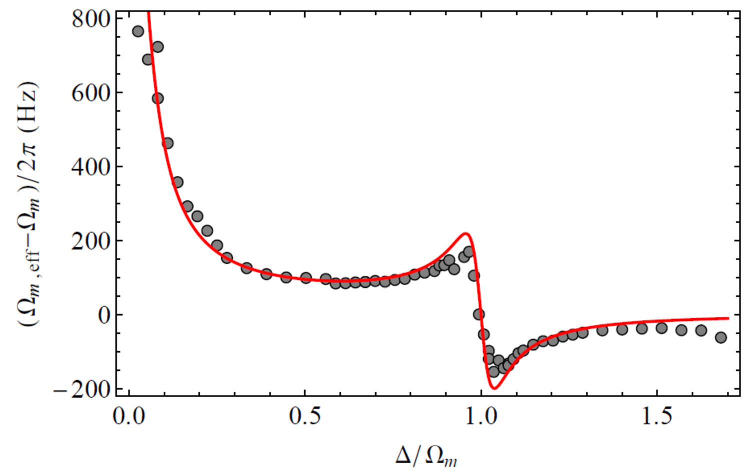
EXPERIMENTAL DATA WITH OUR MEMBRANE



Detected position noise spectrum



Effective damping



Mechanical frequency shift

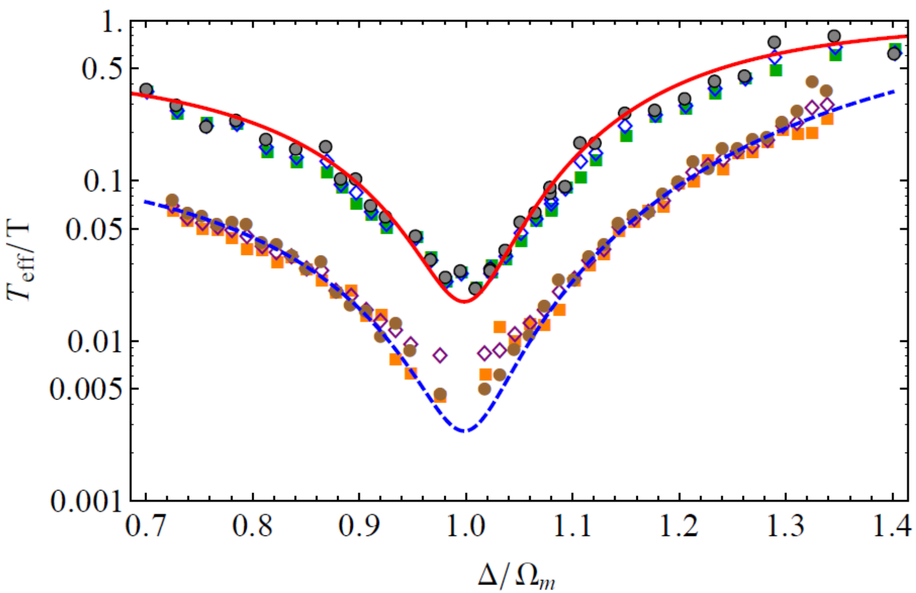
Karuza et al., New J. Phys. 14, 095015 (2012).

2. Significant cooling when $\Delta = \omega_m$

$$\left\langle \delta x^2 \right\rangle = \int \frac{d\omega}{2\pi} S_x(\omega) = \frac{kT_{eff}}{m\Omega_m^2}$$

Effective temperature ∝ area of the resonance peak

Overdamping ⇔ cooling



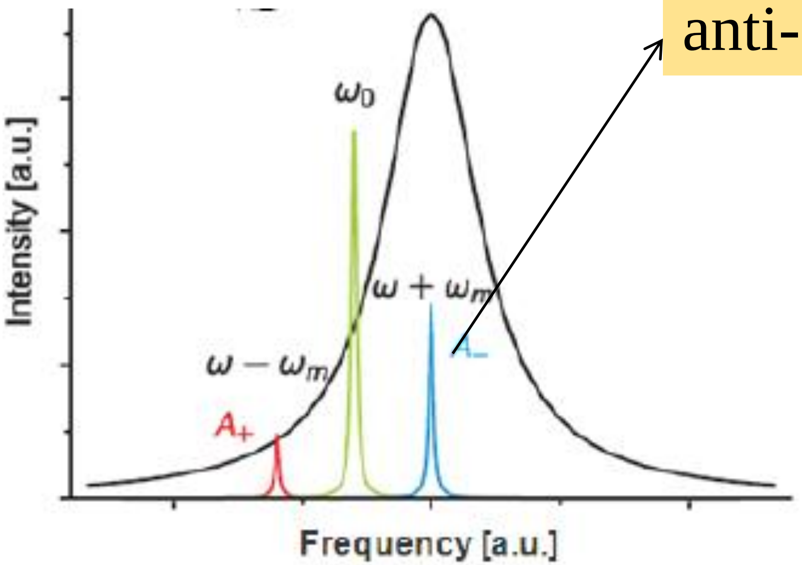
Karuza et al., New J. Phys. 14, 095015 (2012).

Effective resonator temperature

⇒ **membrane mode laser-cooled down to ~ 1 K**

The resonator is cooled by the cavity mode = **effective additional zero-temperature reservoir**, optimally coupled when $\Delta = \omega_m$.

Sideband description of the $\Delta = \omega_m$ resonance



anti-Stokes sideband resonant with the cavity

$$A_{\pm} = \frac{G^2 \kappa}{2\left[\kappa^2 + (\Delta \pm \omega_m)^2\right]}$$

Rates at which photons are scattered by the moving oscillator, yielding absorption (Stokes, A_+) or emission (anti-Stokes, A_-) of vibrational phonons

$$\Gamma = A_- - A_+$$

net laser cooling rate (positive for $\Delta > 0$)

Connection between the two descriptions

$$\gamma_m^{eff}(\omega_m) = \gamma_m + \frac{2G^2 \Delta \omega_m \kappa}{\left|(\kappa - i\omega_m)^2 + \Delta^2\right|^2} \equiv \gamma_m + \Gamma$$

C. Genes, A. Mari, D. Vitali and P. Tombesi, Adv. At. Mol. Opt. Phys., Vol. 57, 2009, pp. 33-86.

With a good approximation $\langle \delta p^2 \rangle \simeq \langle \delta q^2 \rangle = n_{eff} + 1/2$

$$\boxed{n_{eff} = \frac{\gamma_m \bar{n} + A_+}{\gamma_m + \Gamma}}$$

$$\bar{n} = \left[e^{\frac{\hbar\omega_m}{kT}} - 1 \right]^{-1}$$

Ground state cooling achieved when $\Gamma \sim A_- \gg A_+$, which is realized when the blue sideband is resonant with the cavity, $\Delta = \omega_m$ and in the resolved sideband limit $\omega_m \gg \kappa$

$$\boxed{n_{eff} \approx \frac{A_+}{A_-} \approx \frac{\kappa^2}{4\omega_m^2} << 1}$$

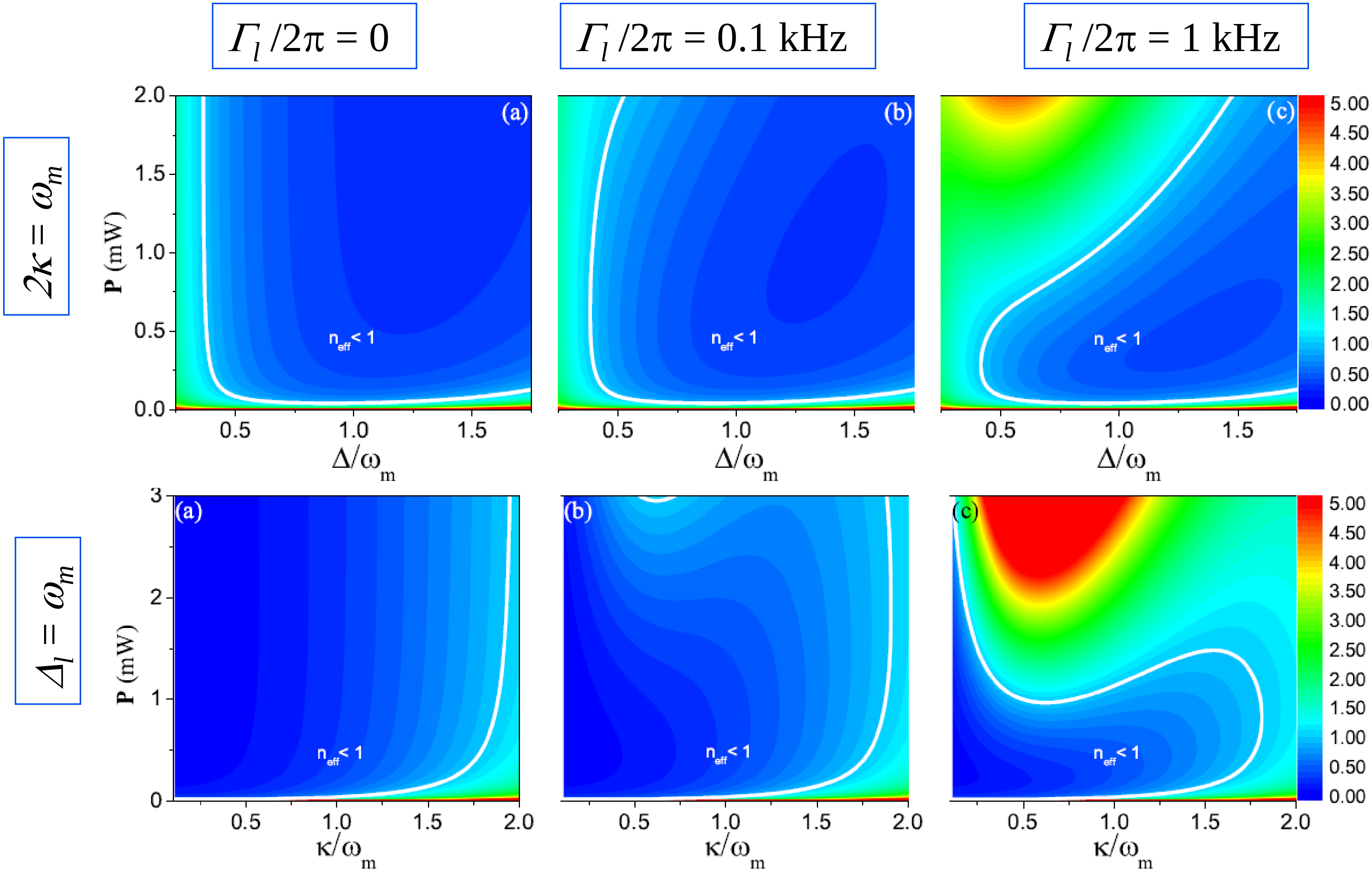
Effect of driving laser frequency noise

$$\boxed{n_{\text{eff}} = \frac{1}{\gamma_{\text{m}} + \Gamma_{\text{op}}} \left[n\gamma_{\text{m}} + A_+ + \frac{|\alpha_s|^2 \Delta \Gamma_{\text{op}}}{2\kappa\omega_{\text{m}}} S_{\dot\phi}(\omega_{\text{m}}^{\text{eff}}) \right]}$$

$\boxed{S_{\dot\phi}(\omega_{\text{m}}^{\text{eff}})}$ **Frequency noise at resonance must be small enough** for achieving the quantum ground state

M. Abdi et al., PRA 84, 032325 (2011)

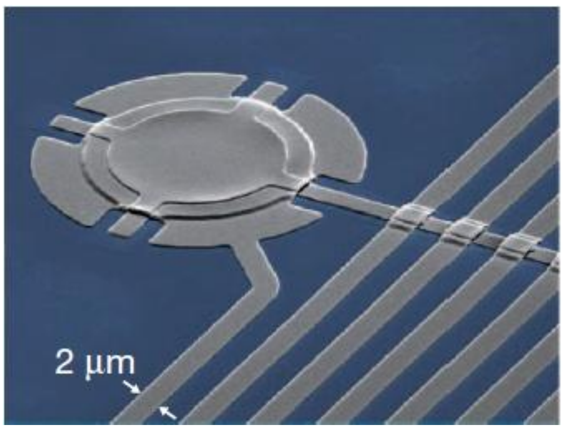
THEORY PREDICTIONS ON COOLING WITH LASER NOISE



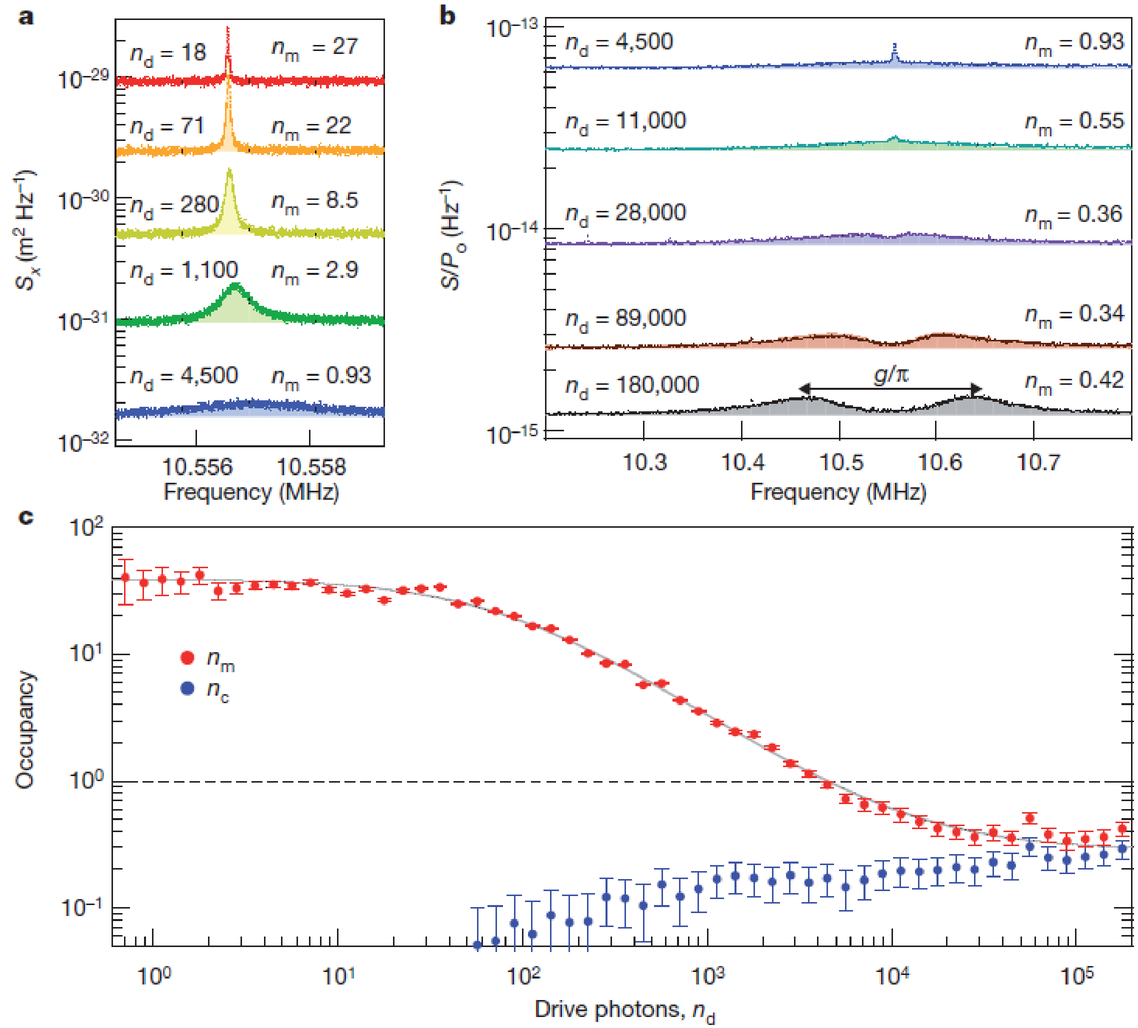
State-of-art experimental parameters: $m=10$ ng, $\omega_m/2\pi = 10$ MHz, $\gamma_m/2\pi = 5$ Hz, $G_0 = 1$ kHz, $L = 1$ mm, $T = 400$ mK, $\Omega/2\pi = 50$ kHz, $\gamma = \Omega/2$

CAVITY COOLING TO GROUND STATE ACHIEVED

1. **micromechanical membrane embedded into a superconducting** **microwave resonant LC circuit**

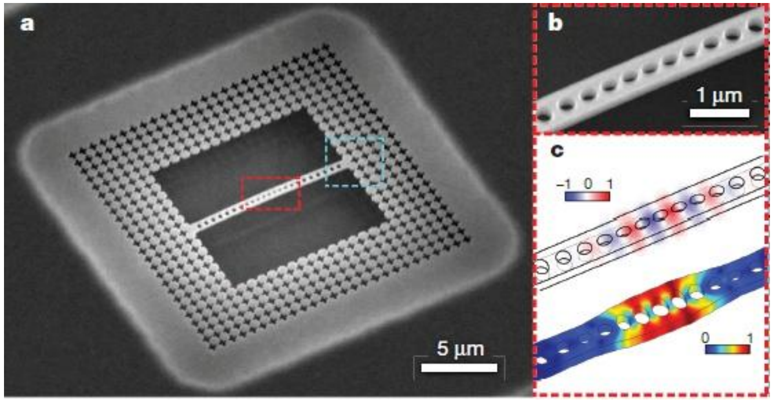


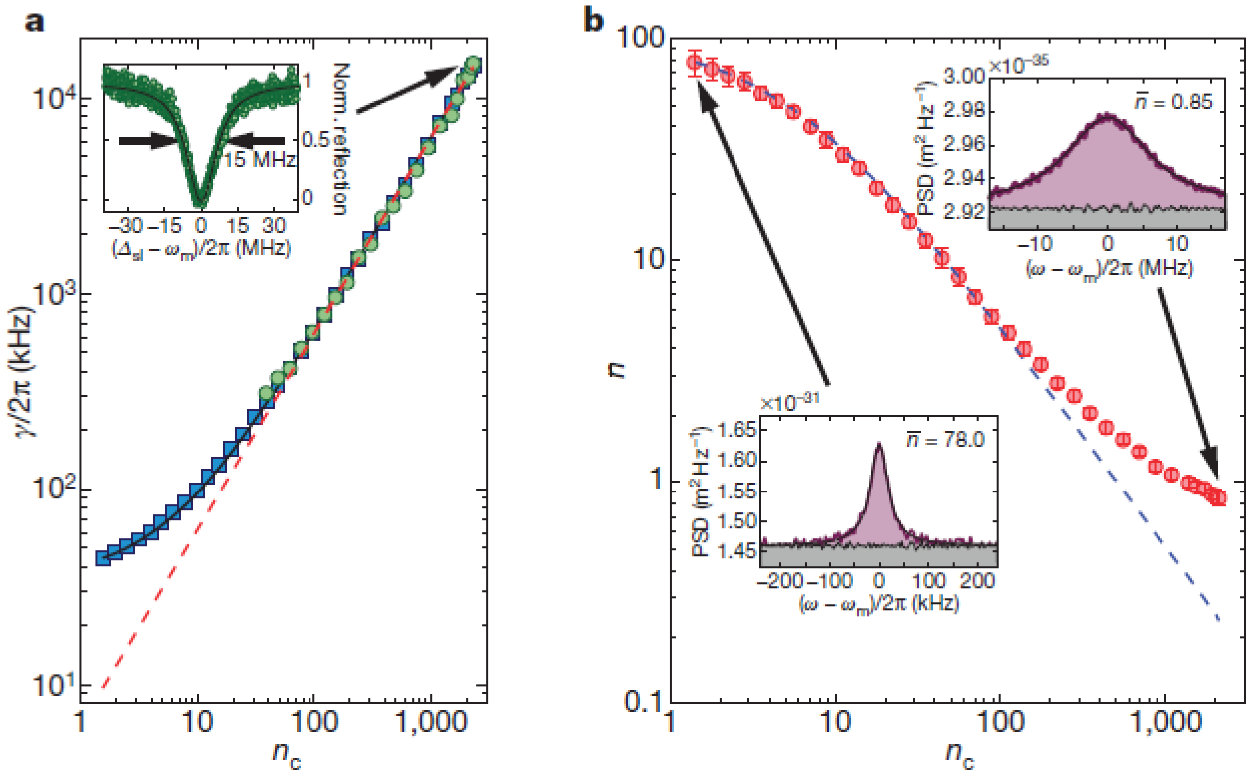
J.D. Teufel et al. Nature 475, 359 (2011)



Thermalization to the effective temperature of the microwave cavity (at 10 mK)

2. Patterned silicon nanobeam with an acoustic resonance (**breathing mode**), which is **coupled by radiation pressure to the co-localized optical resonance** (**photonic crystal zipper cavity)**



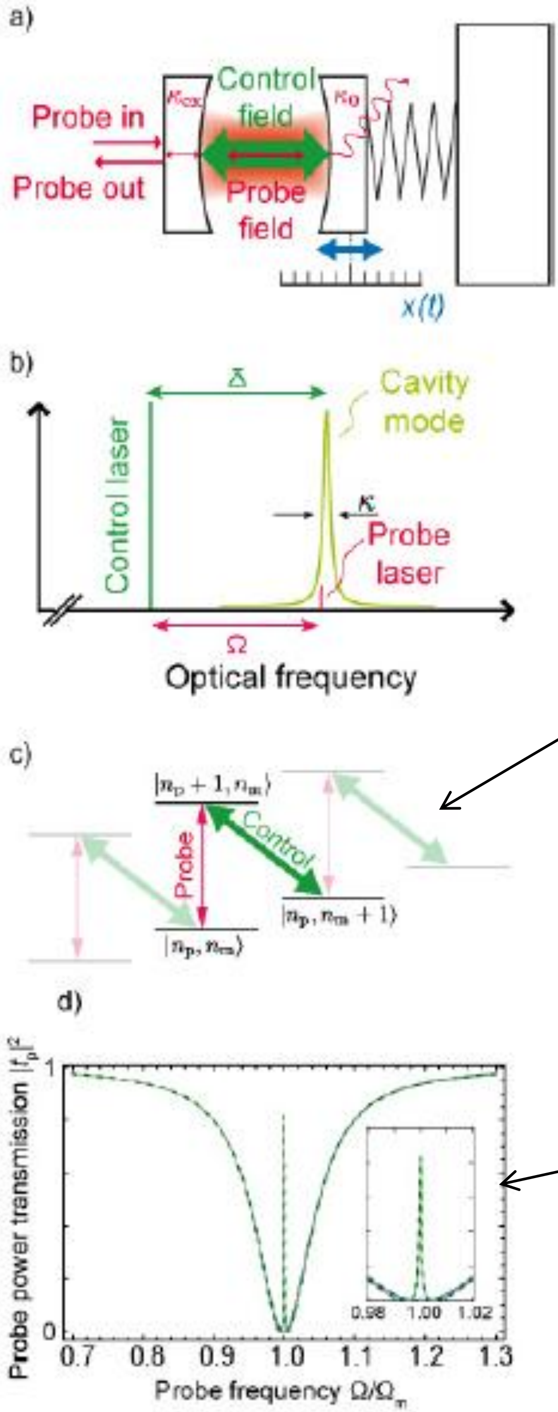


J. Chan et al., Nature 478, 89 (2011)

EFFECT OF THE MECHANICAL RESONATOR ON THE OPTICAL FIELD

OPTOMECHANICALLY INDUCED TRANSPARENCY (OMIT)

The optomechanical analogue of electromagnetically-induced transparency (EIT) (“riga nera” 1976, Gozzini et al.)



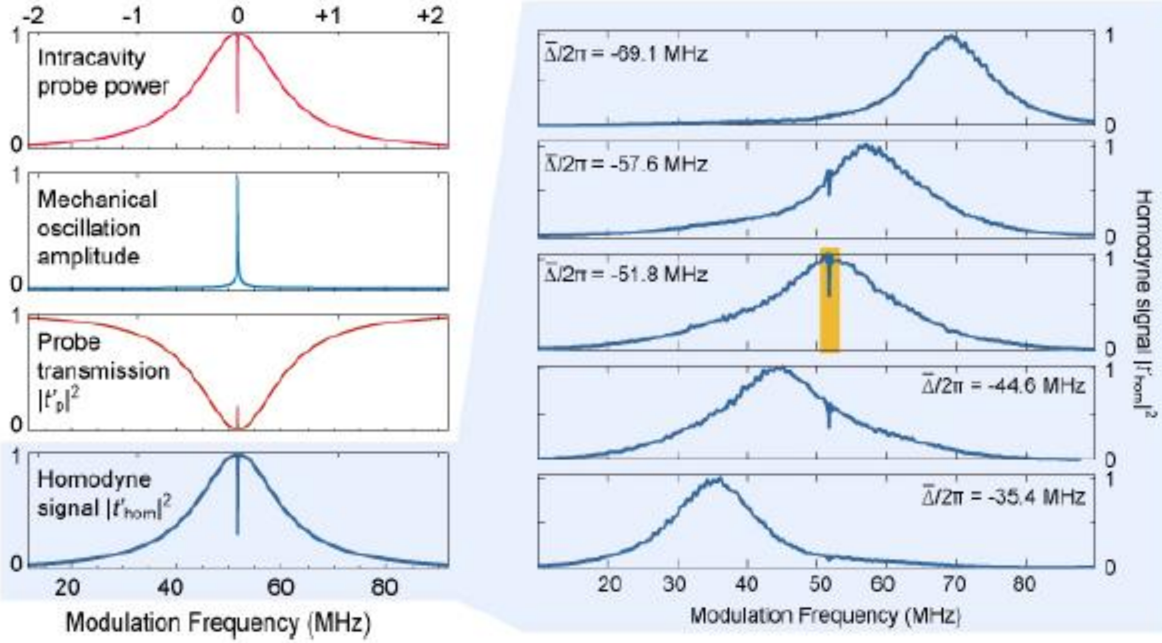
The optomechanical analogue of EIT occurs when

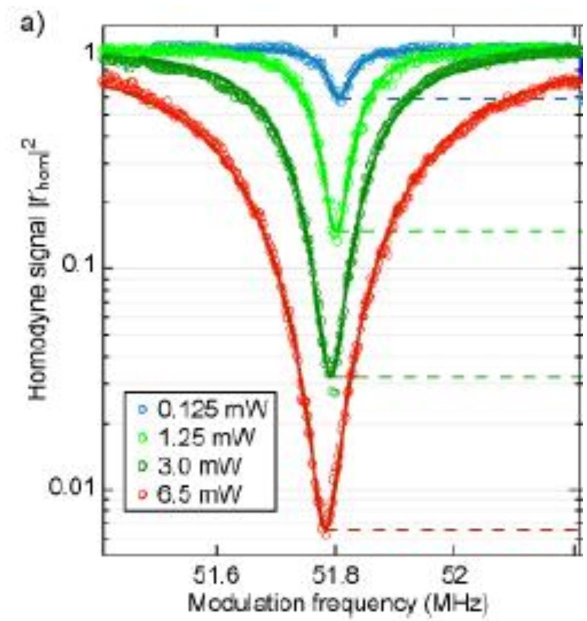
1. **an additional weak probe field is sent into the cavity**
2. the optimal cooling condition is verified, i.e., the **blue sideband of the laser is resonant with the cavity, $\Delta = \omega_m$**

Agarwal & Huang, PRA 2010

The probe at resonance is perfectly transmitted by the cavity instead of being fully absorbed: **destructive interference between the probe and the anti-Stokes sideband of the laser**

Weis et al, *Science* ***330,*** 1520 (2010).





Width of the transparency window

$$\boxed{\begin{aligned} \gamma_m^{eff} &= \gamma_m + \Gamma \\ \gamma_m^{eff} &= \gamma_m (1 + C) \end{aligned}}$$

when $\Delta = \omega_m$

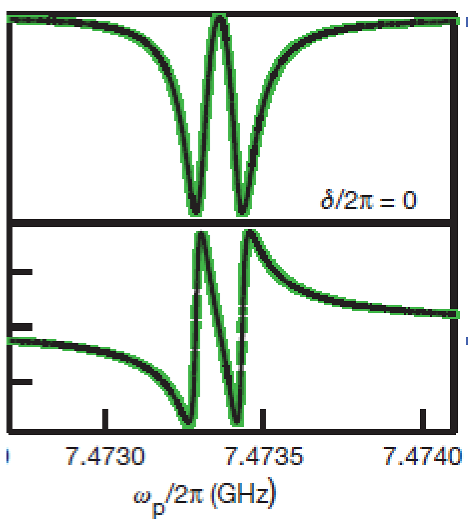
$$\boxed{C = G^2/2\kappa_{\mathrm{T}}\gamma_{\mathrm{m}}}$$

cooperativity

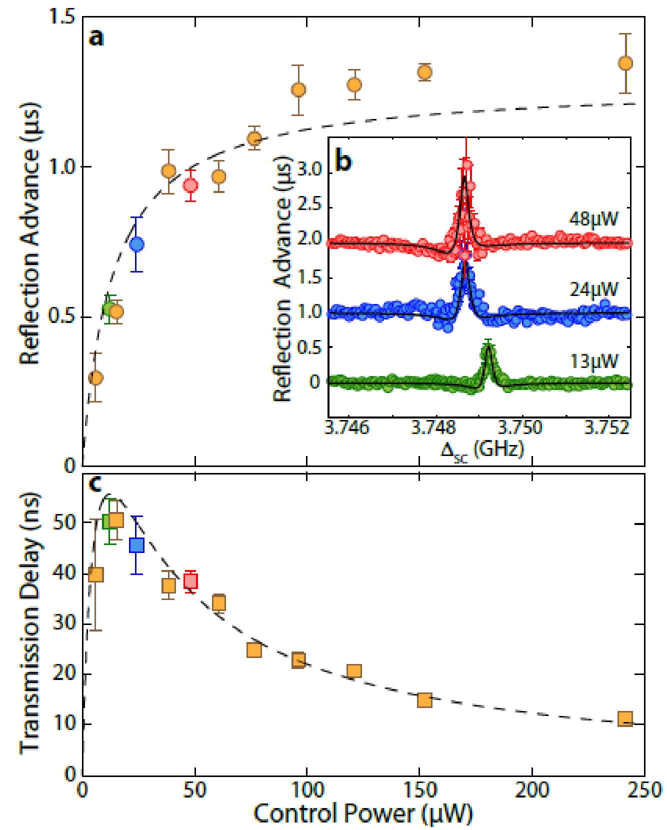
EIT is negligible out of the resonant condition $\Delta = \omega_m$

Concomitant with transparency, one has **strong group dispersion ⇔ slow light and superluminal effects**

⇒ EIT can be used for **tunable optical delays**, for stopping, storing and retrieving classical and quantum information



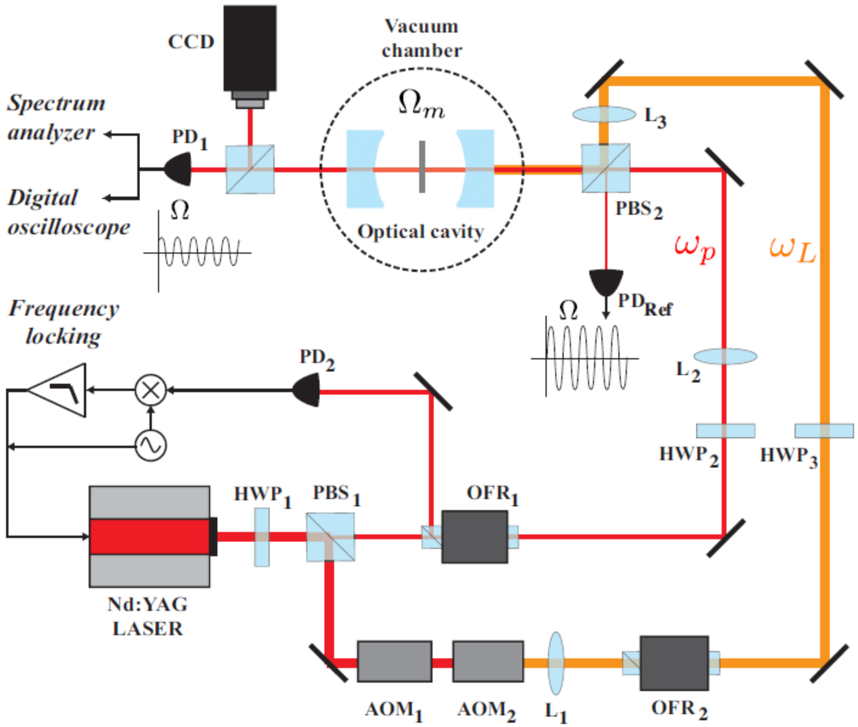
Teufel et al., Nature 471, 204 (2011) (OMIT with an electromechanical system)



A. H. Safavi-Naeini et al., Nature (London) 472, 69 (2011).

OUR OMIT EXPERIMENT WITH THE MEMBRANE

1. **Room temperature**
2. **Significantly lower frequencies (~ 350 kHz) rather than Ghz**
   $\Rightarrow$ longer delay times
3. **Free space (rather than guided) optics**

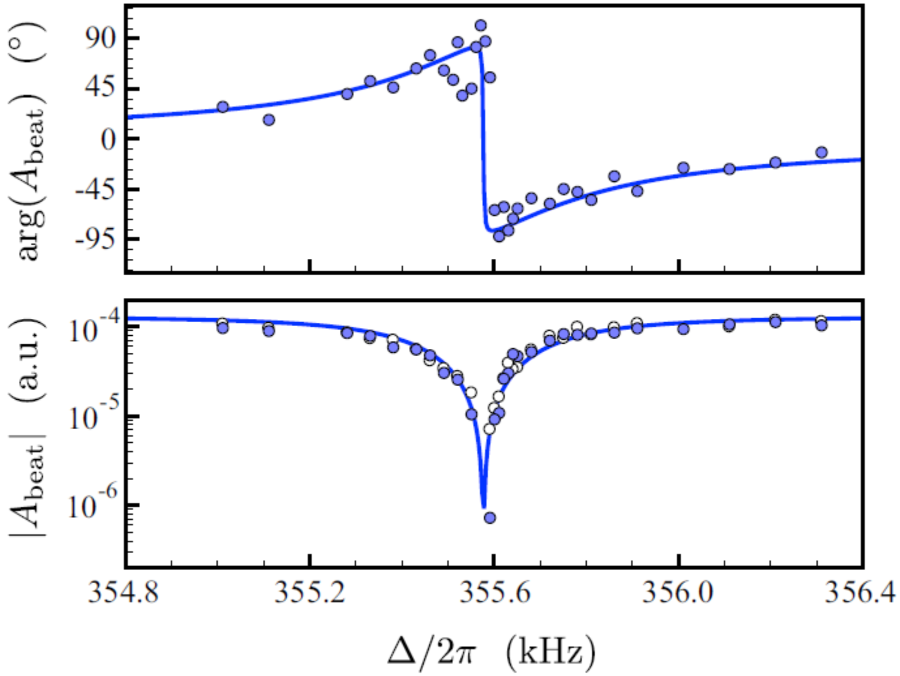


OMIT versus atomic EIT

1. it does not rely on naturally occurring resonances $\Rightarrow$ applicable to **inaccessible wavelengths;**
2. a single optomechanical element can already achieve unity contrast
3. **Long optical delay times achievable**, since they are limited only by the mechanical decay time

Karuza et al., arXiv:1209.1352v1

MEASURED PHASE AND AMPLITUDE OF THE TRANSMITTED BEAM



(we have induced "opacity" rather than transparency)

Estimated **max group delay τ = 670 ns** (from the derivative of the phase shift)

$$A_{\text{beat}} = \frac{4\kappa_2\kappa_0|s_\text{p}|}{\kappa_\text{T}}\sqrt{\frac{\mathcal{P}}{\hbar\omega_\text{L}\left(\kappa_\text{T}^2+\Delta^2\right)}}\left[1+\text{i}\frac{G^2\chi_\text{eff}(\Delta)}{2\kappa_\text{T}}\right]$$

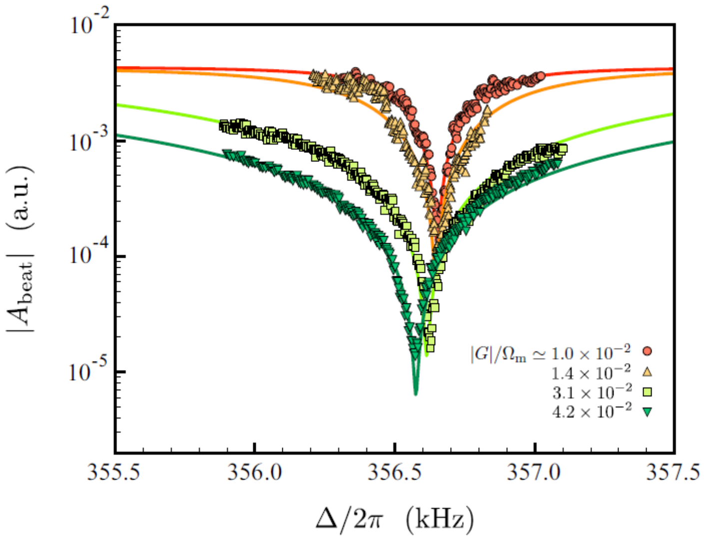
$$\tau_{\max}^{\text{R}} = \frac{2}{\gamma_m}\frac{\eta C}{(1+C)(1-\eta+C)}$$

$$C = G^2/2\kappa_\text{T}\gamma_\text{m}$$

Cooperativity (~160 here)

$$\eta = \frac{\kappa_{ex}}{\kappa_T}$$

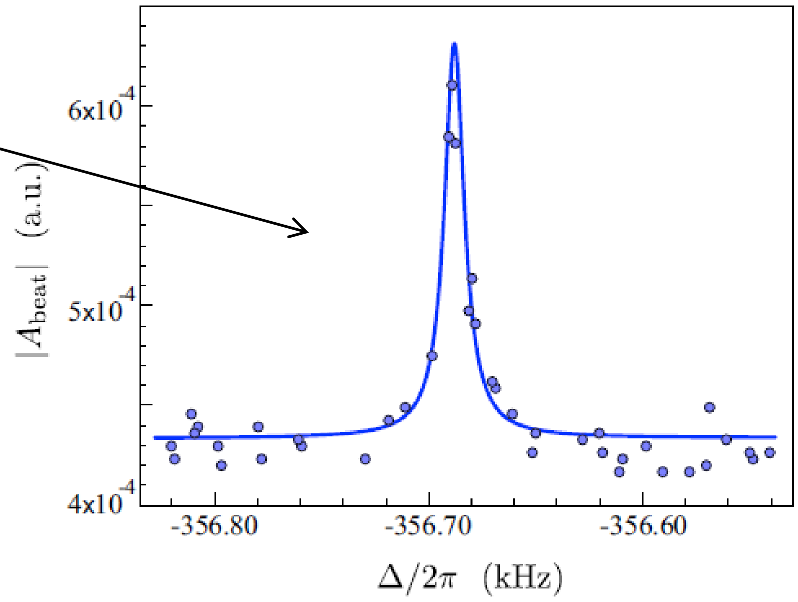
Karuza et al., arXiv:1209.1352v1



The delay and the transparency window are here **tunable by shifting the membrane**, without varying power

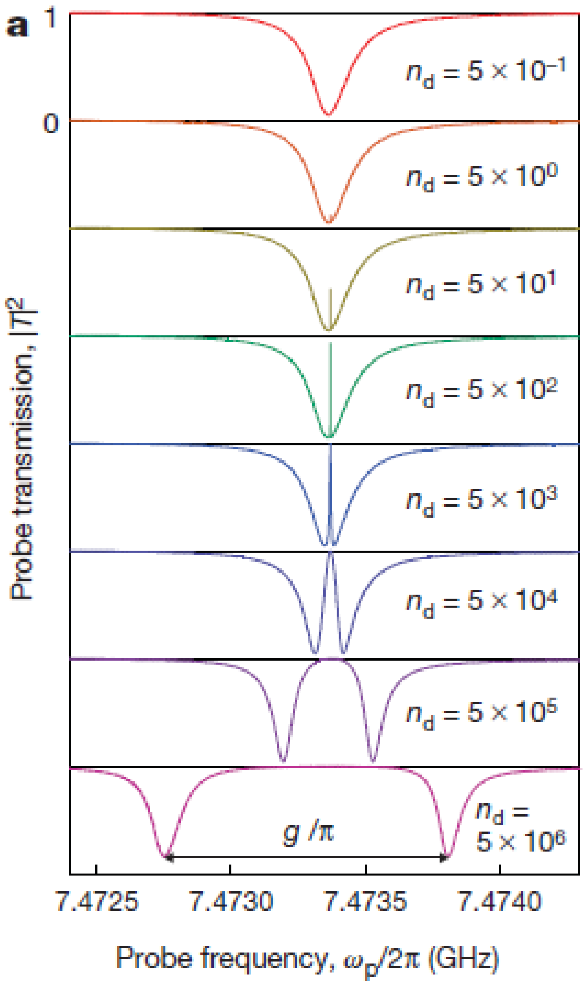
When the **red** sideband of the laser is resonant with the cavity, $\boldsymbol{\Delta = -\omega_m}$, one has instead **constructive interference** and **"optomechanically induced amplification"**

One has the **optomechanical analogue of a parametric oscillator below threshold**



Karuza et al., arXiv:1209.1352v1

EIT also observed with microwave signals in an electromechanical system



Progressive transition from EIT to normal-mode splitting for increasing driving power ⇔ increasing coupling)

Normal-mode splitting ⇔ appearence of **hybidized "optomechanical" eigenmodes**, with mixed photonic and phononic nature

Teufel et al., Nature 471, 204 (2011)

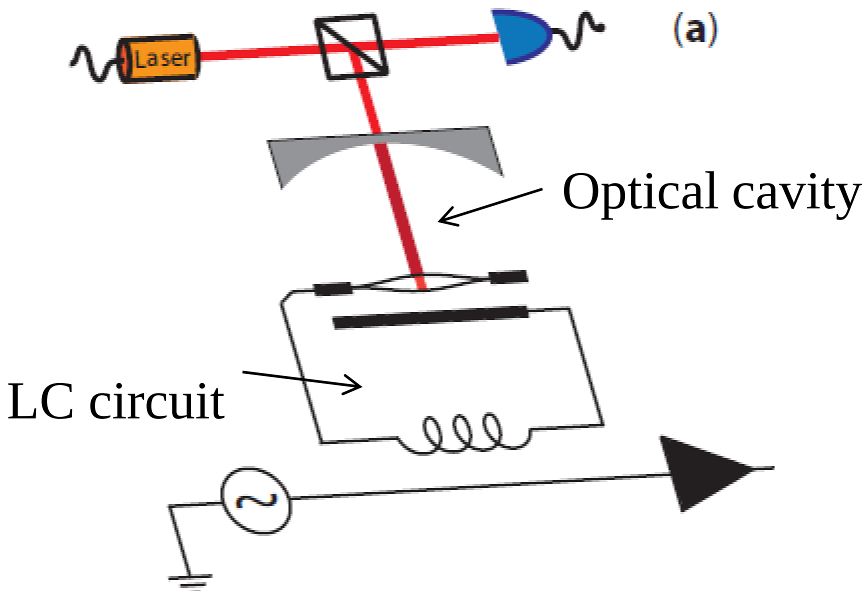
How to use a nanomechanical resonator as a quantum interface between optics and microwaves

Based on:

1. Establishing **strong continuous variable (CV) entanglement** between the optical and microwave output field
2. High-fidelity CV optical-to-microwave **teleportation** of nonclassical states

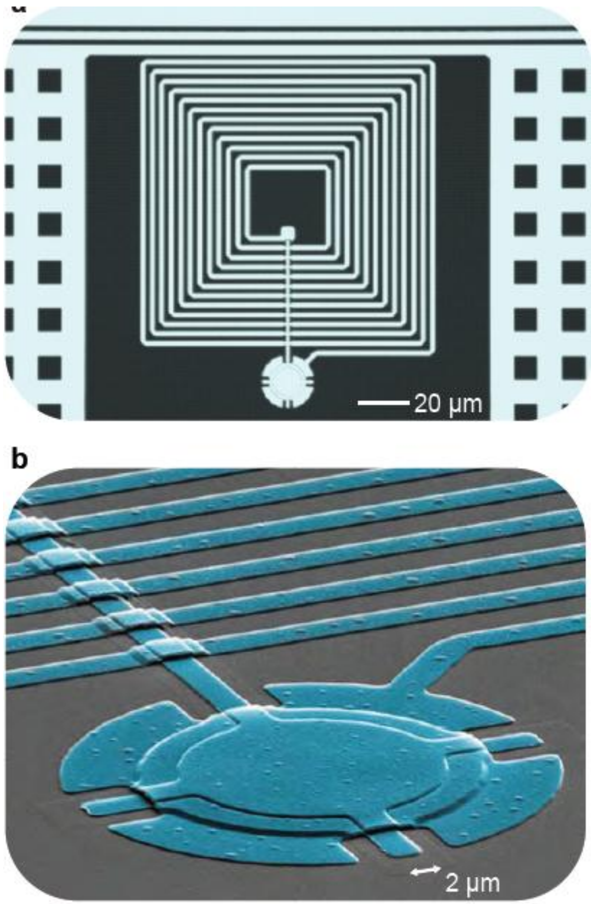
S. Barzanjeh, M. Abdi, G.J. Milburn, P. Tombesi, D. Vitali,
Phys. Rev. Lett. 109, 130503 (2012).

Light is optimal for quantum communications between nodes, while microwave are often used for manipulating solid state quantum processors $\Rightarrow$ **a quantum interface between optical and microwave photons would be extremely useful**

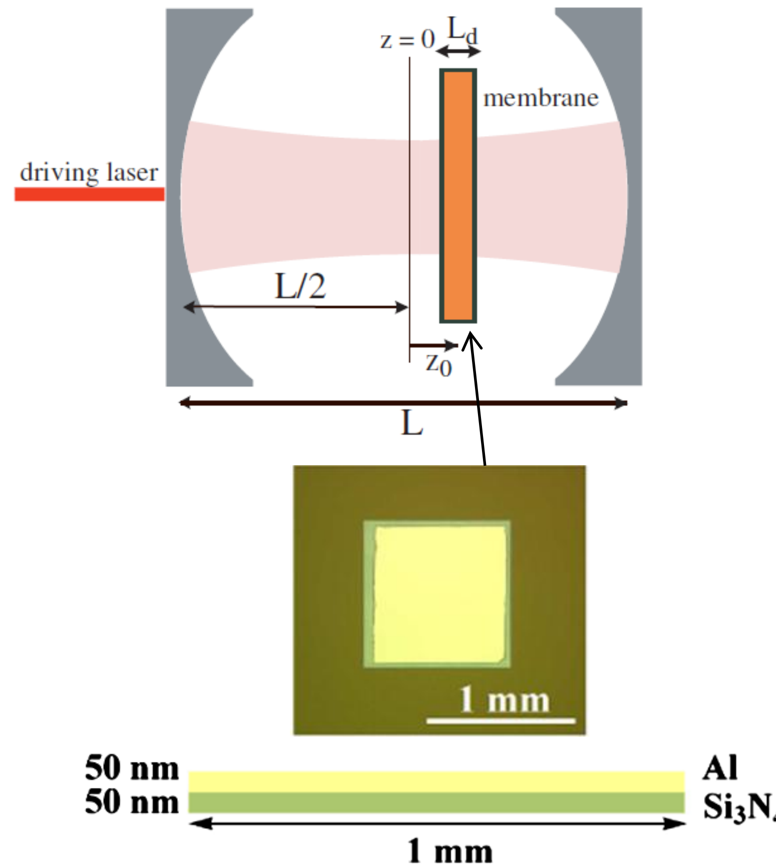


We propose a quantum interface between optical and microwave photons based on a **nanomechanical resonator in a superconducting circuit, simultaneously interacting with the two fields**

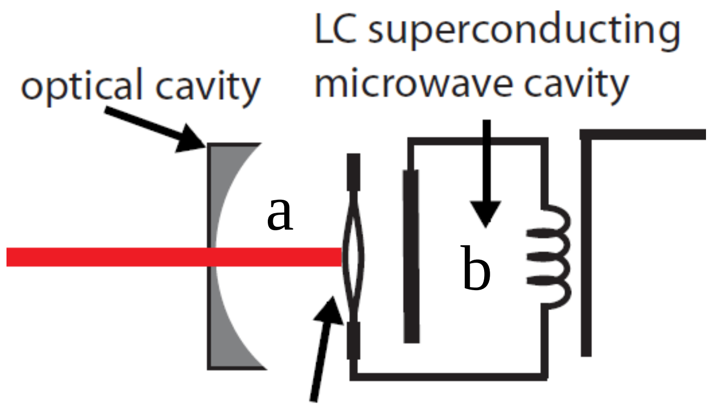
The **membrane resonator is coupled capacitively with the microwave cavity and by radiation pressure with the optical cavity. Possible implementations**:



Adding an optical cavity by coating the membrane capacitor of the superconducting LC circuit of Teufel et al., Nature (London), 471, 204 (2011).



Adding a LC circuit to the membrane-in-the-middle setup with a patterned Al film on top, Yu et al PRL 108, 083603 (2012)



Both cavities are driven coherently: $\Rightarrow$ the dynamics of the **quantum fluctuations around the stable steady state well described by Quantum Langevin Equations (QLE)**

The nanomechanical resonator **mediates a retarded interaction between the two cavity fields (exact QLE), with a kernel** $\boxed{\chi_M(t) = e^{-\gamma_m t/2} \sin \omega_m t}$

$$\begin{aligned}
\delta\dot{\hat{a}} &= -\kappa_c \delta\hat{a} + \sqrt{2\kappa_c}\hat{a}_{in}(t)e^{i\Delta_c t} + \frac{i}{2}\int_{-\infty}^{t} ds \chi_M(t-s)\left\{G_c \hat{\xi}(s)e^{i\Delta_c t}\right. \\
&\left. + G_c^2\left[\delta\hat{a}(s)e^{i\Delta_c(t-s)} + \delta\hat{a}^\dagger(s)e^{i\Delta_c(t+s)}\right] + G_c G_w\left[\delta\hat{b}(s)e^{i\Delta_c t - i\Delta_w s} + \delta\hat{b}^\dagger(s)e^{i\Delta_c t + i\Delta_w s}\right]\right\}, \\
\delta\dot{\hat{b}} &= -\kappa_w \delta\hat{b} + \sqrt{2\kappa_w}\hat{b}_{in}e^{i\Delta_w t} + \frac{i}{2}\int_{-\infty}^{t} ds \chi_M(t-s)\left\{G_w \hat{\xi}(s)e^{i\Delta_w t}\right. \\
&\left. + G_w^2\left[\delta\hat{b}(s)e^{i\Delta_w(t-s)} + \delta\hat{b}^\dagger(s)e^{i\Delta_w(t+s)}\right] + G_c G_w\left[\delta\hat{a}(s)e^{i\Delta_w t - i\Delta_c s} + \delta\hat{a}^\dagger(s)e^{i\Delta_w t + i\Delta_c s}\right]\right\}
\end{aligned}$$

Beamsplitter-like optical-microwave interaction $\Rightarrow$ **state transfer term**

parametric optical-microwave interaction $\Rightarrow$ **entangling term**

One can resonantly select one of these processes by appropriately adjusting the two cavity detunings:

• Equal detunings: **$\Delta_c = \Delta_w \Rightarrow$ state transfer** between optics and microwave (see other proposals, Tian et al., 2010, Taylor et al., PRL 2011, Wang & Clerk, PRL 2011)

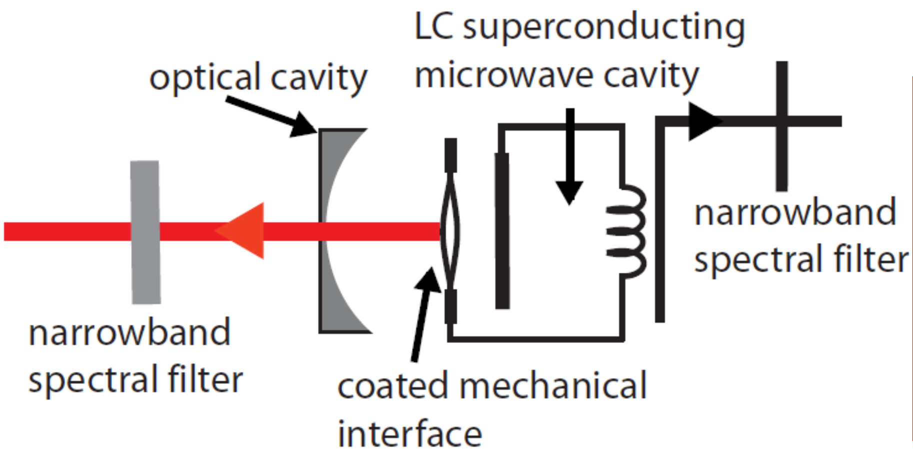
• **Opposite detunings: $\Delta_c = -\Delta_w \Rightarrow$ two-mode squeezing and entanglement**

Here we choose $\Delta_c = -\Delta_w = \pm\,\omega_m \Rightarrow$ two-mode squeezing and entanglement is resonantly enhanced (because the interaction kernel does not average to zero)

The mechanical interface realizes an **effective parametric oscillator with an optical signal (idler) and microwave idler (signal)** $\Leftrightarrow$ microwave-optical two mode squeezing

Similarly to single-mode squeezing, **two mode squeezing can be very strong for the OUTPUT cavity fields**

by properly choosing the central frequency Ω_j and the bandwidth $1/\tau$ of the output modes, one can optimally filter the entanglement between the two output modes



output cavity modes

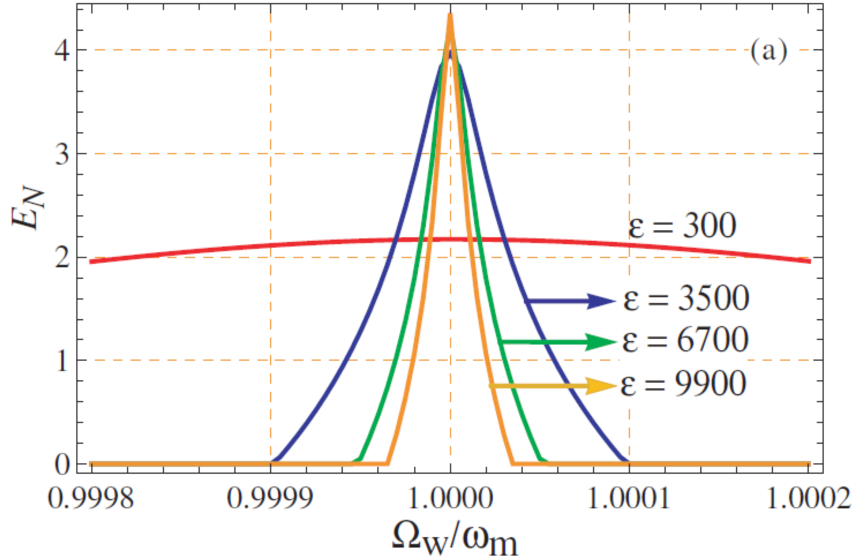
$$\hat{a}_c^{out}(t) = \int_{\infty}^{t} ds g_c(t-s)\hat{a}^{out}(s)$$

$$\hat{b}_w^{out}(t) = \int_{\infty}^{t} ds g_w(t-s)\hat{b}^{out}(s)$$

normalized causal filter function

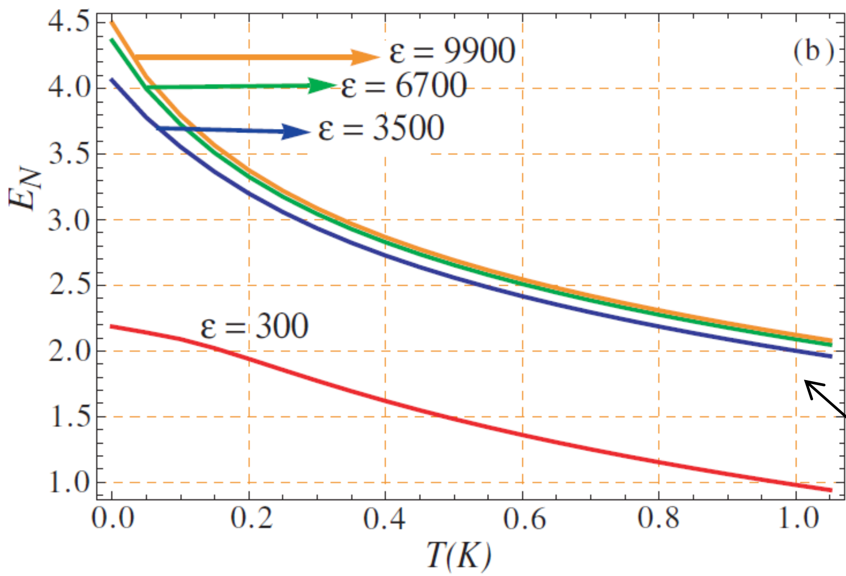
$$g_j(t) = \sqrt{\frac{2}{\tau}}\theta(t)e^{-(1/\tau+i\Omega_j)t} \quad j = c, w$$

OUTPUT MICROWAVE-OPTICAL ENTANGLEMENT



LARGE ENTANGLEMENT FOR **NARROW-BAND OUTPUTS**

LogNeg at four different values of the normalized inverse bandwidth $\epsilon = \tau\omega_m$ *vs* the normalized frequency Ω_w/ω_m, at fixed central frequency of the optical output mode $\Omega_c = -\omega_m$.

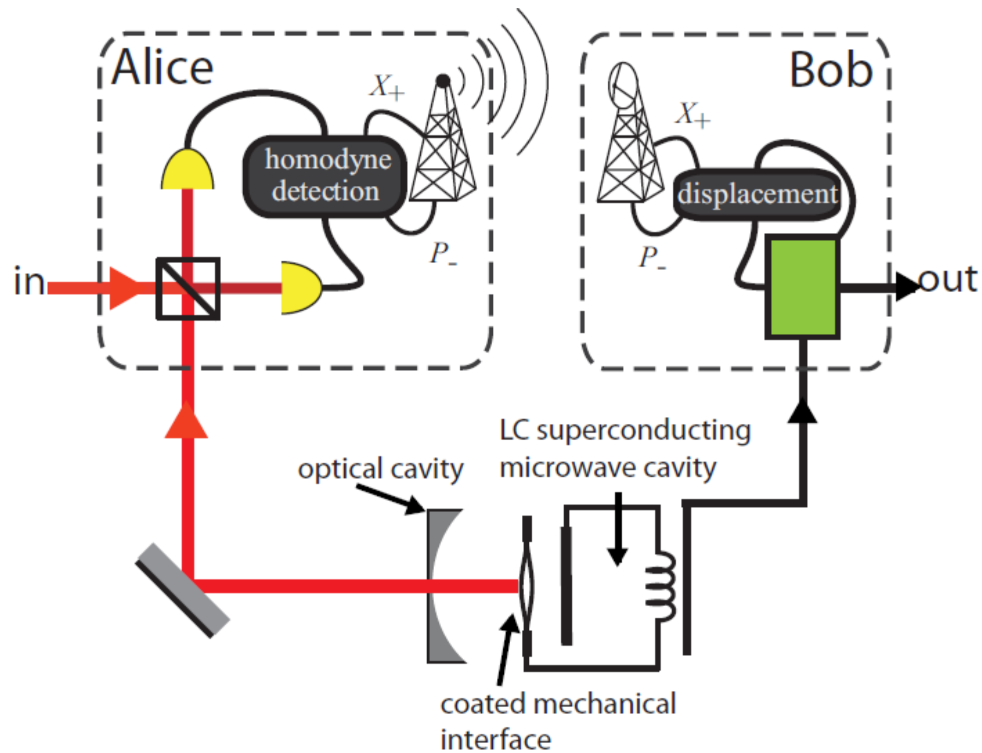


Optical and microwave cavity detunings fixed at $\Delta_c = -\Delta_w = -\omega_m$
Other parameters: $\omega_m/2\pi = 10$ MHz, $Q=1.5\text{x}10^5$, $\omega_w/2\pi = 10$ GHz, $\kappa_w = 0.04\omega_m$, $P_w = 42$ mW, $m = 10$ ng, $T = 15$ mK. This set of parameters is analogous to that of Teufel et al.
Optical cavity of length $L = 1$ mm and damping rate $\kappa_c = 0.04\omega_m$, driven by a laser with power $P_c = 3.4$ mW.

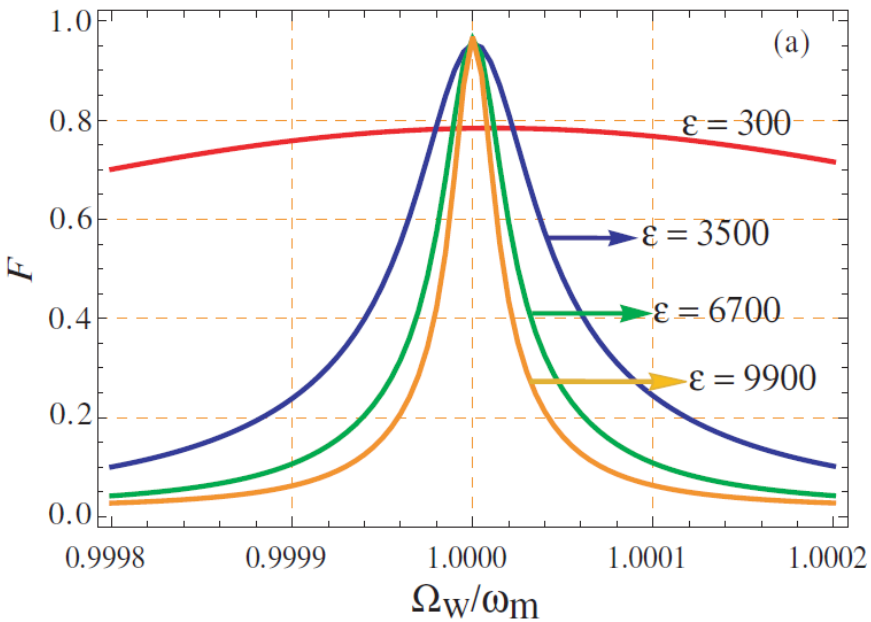
Entanglement is robust wrt to temperature

- The common interaction with the nanomechanical resonator establishes **quantum correlations which are strongest between the output Fourier components *exactly at resonance* with the respective cavity field**

Such a large stationary entanglement can be exploited for **continuous variable (CV) optical-to-microwave quantum teleportation**:

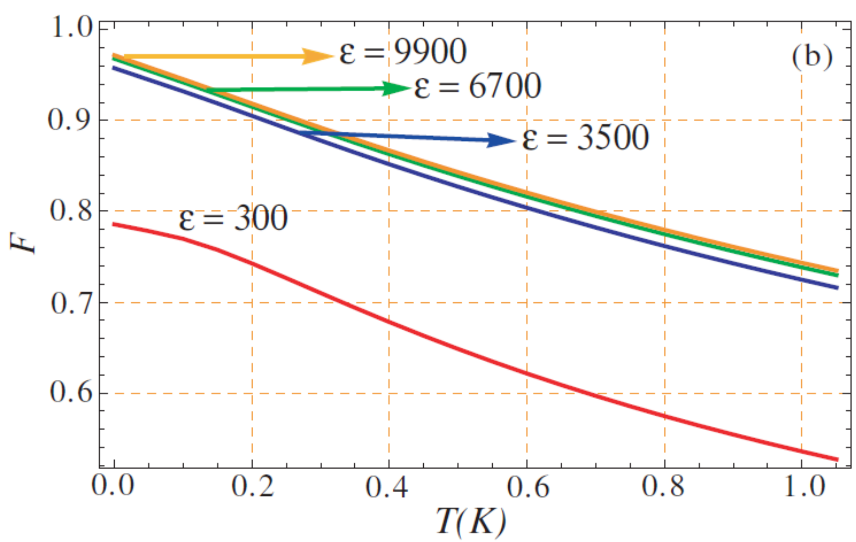


TELEPORTATION FIDELITY OF A CAT STATE



Input cat state

$$|\psi\rangle = N(|\alpha\rangle + |-\alpha\rangle)$$

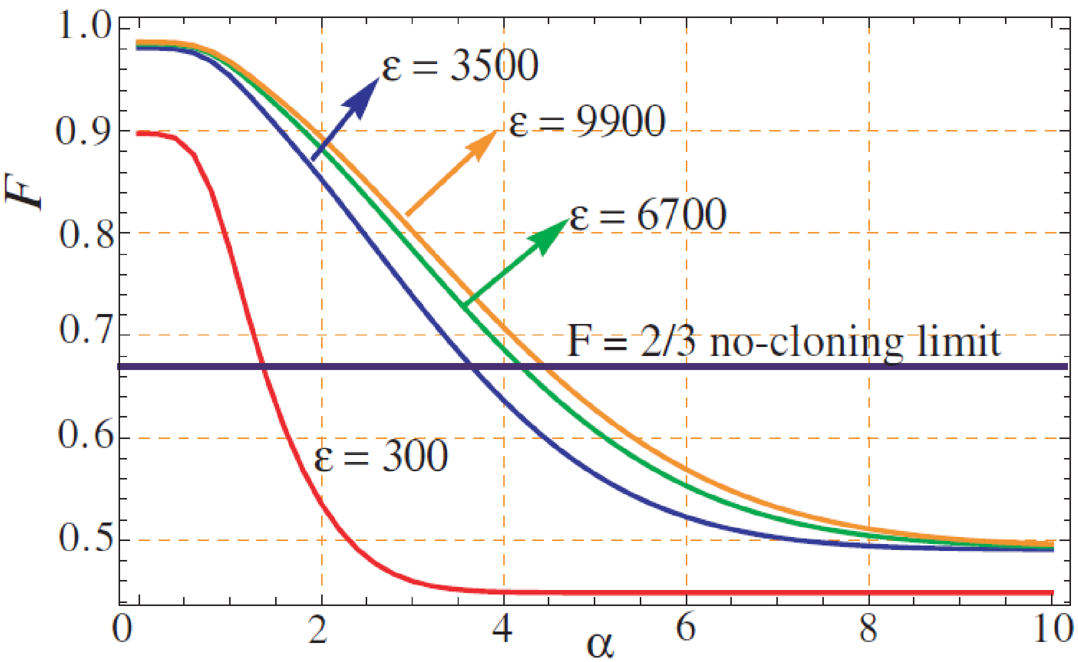


(a) Teleportation fidelity F at four different values of $\epsilon = \tau\omega_m$ *versus* Ω_w/ω_m and for the Schrodinger cat-state amplitude $\alpha = 1$.

(b) Plot of F for the same values of ϵ *vs* temperature at a fixed central frequency of the microwave output mode $\Omega_w = \omega_m$.

The fidelity behaves as the logneg

TELEPORTATION FIDELITY OF NONCLASSICALITY



- the selected narrow-band microwave and optical output modes possess (EPR) correlations that can be optimally exploited for teleportation

- ***F* is very close to the optimal upper bound achievable for a given E_N**

$$\boxed{F_{opt} = \frac{1}{1+e^{-E_N}}}$$

Through teleportation we realize a high-fidelity optical-to-microwave quantum state transfer assisted by measurement and classical communication

CONCLUSIONS

Micro- and nano-mechanical resonators are promising candidates for quantum and classical interfaces

1. One can **cool to the ground state** the mechanical oscillator

2. **One can control optomechanical interference effects like EIT and operate tunable delays**

3. Optics-to-microwave interfaces at the classical and quantum level can be realised

Review paper: C. Genes, A. Mari, D. Vitali and P. Tombesi, *Quantum Effects in Optomechanical Systems*, Advances in Atomic, Molecular, and Optical Physics, Vol. 57, Academic Press, 2009, pp. 33-86.

STATIONARY OPTOMECHANICAL ENTANGLEMENT

Continuous variable entanglement at the steady state between optical field quadrature and position and momentum of the resonator

Stationarity $\Leftrightarrow$ infinite lifetime $\Leftrightarrow$ extremely robust entanglement

OPTOMECHANICAL ENTANGLEMENT OF THE STEADY STATE

The correlation matrix *V* provides a quantitative measure of entanglement: **Logarithmic negativity, E_N**

$$V = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

A, B, C = 2x2 sub-matrices

$$E_{\mathcal{N}} = \max[0, -\ln 2\eta^-]$$

$$\eta^{\pm} \equiv 2^{-1/2}\left[\Sigma(V) \pm \left(\Sigma(V)^2 - 4\det V\right)^{1/2}\right]^{1/2}$$

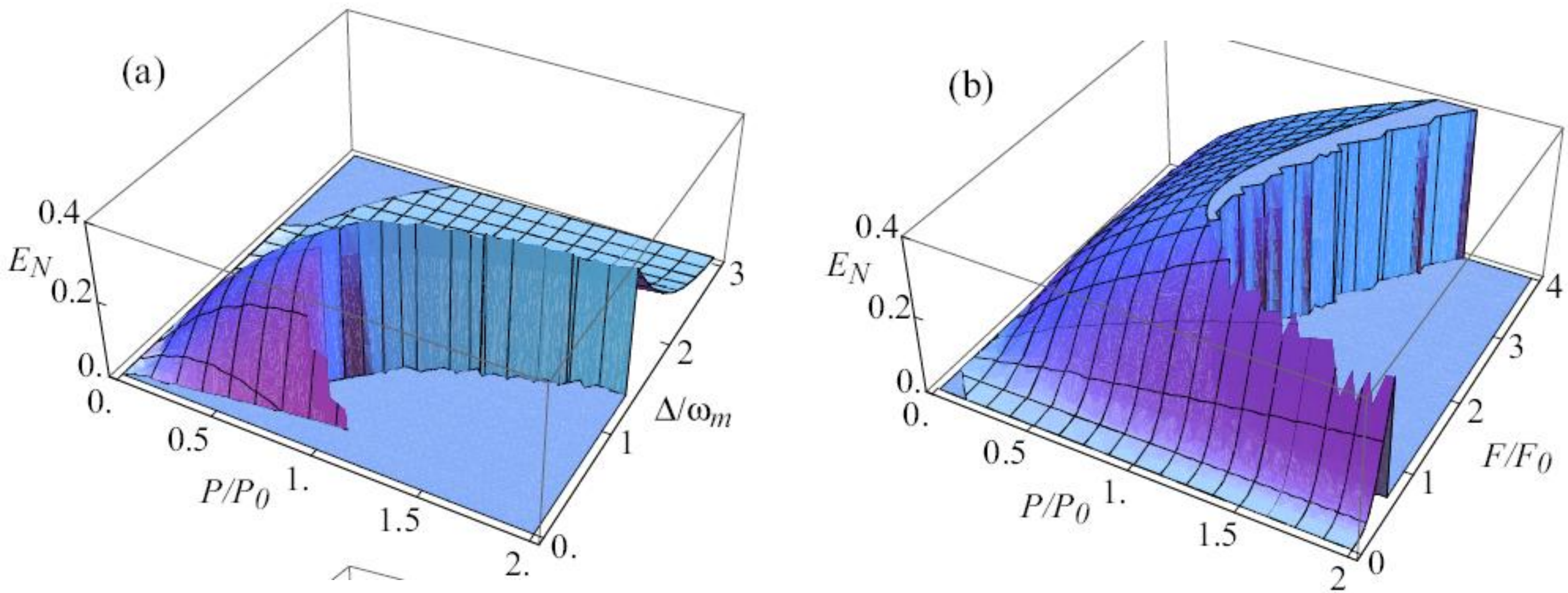
$$\Sigma(V) = \det A + \det B - 2\det C$$

$E_N > 0$ necessary and sufficient condition for entanglement

Not yet experimentally observed, even if achievable with state-of-the-art apparata: **it requires much stronger coupling and it is more sensitive to laser frequency noise**

V can be evaluated by Fourier transforming the quantum Langevin equations and integrating the resulting spectra

Results with no phase noise (D. Vitali et al., PRL 2007)



Parameters: m=10 ng, $\omega_m/2\pi$ = 10 MHz, $\gamma_m/2\pi$ = 100 Hz, P_0 = 50 mW, L = 1 mm, $F_0 = 1.5 \times 10^4$, T = 400 mK

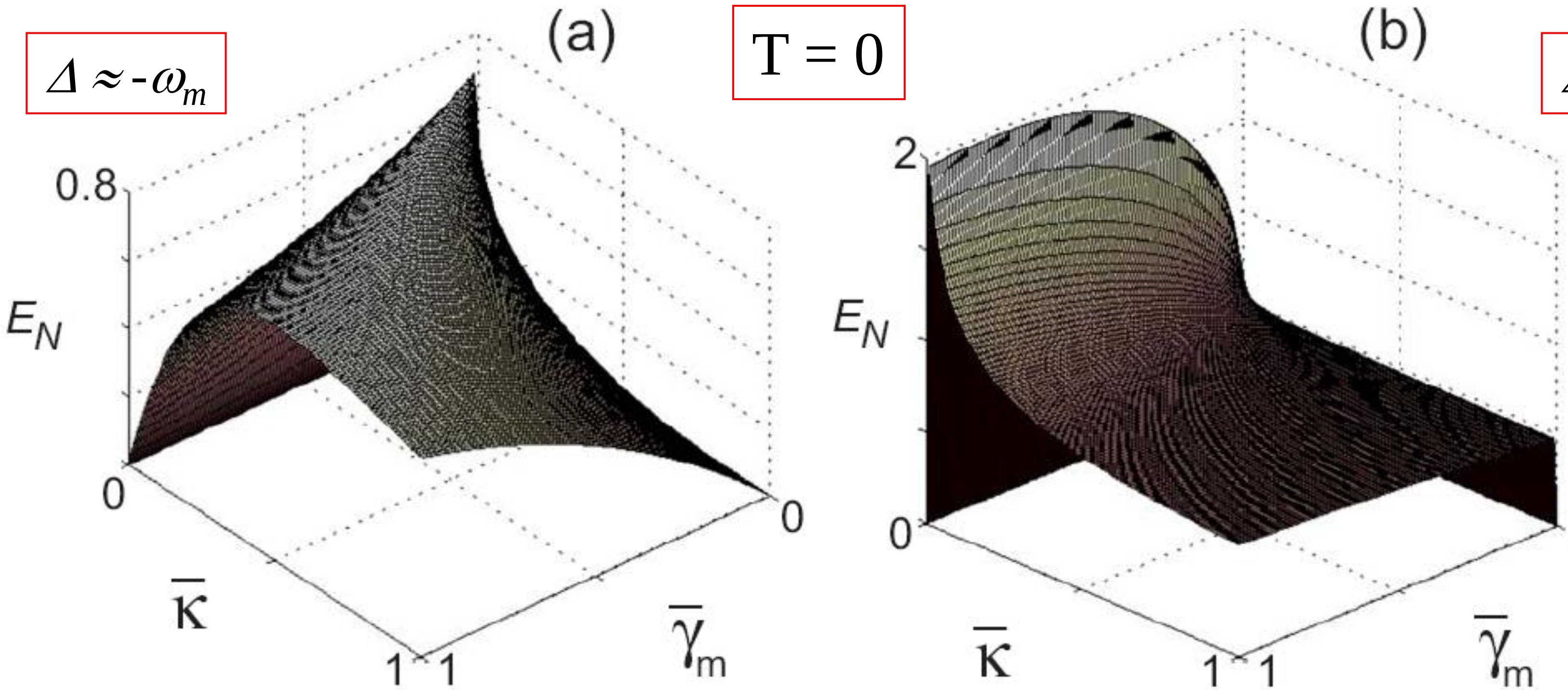
(max entanglement at the instability threshold, where plot interrupts)

Entanglement and cooling are optimal when the cavity is resonant with the antiStokes sideband $\Delta \approx \omega_m$, but entanglement is present (but smaller) **also when $\Delta \approx -\omega_m$**, i.e., resonance with the **Stokes sideband**

$\Delta \approx -\omega_m$

$T = 0$

$\Delta \approx \omega_m$



$\bar{\kappa} = \kappa \times 10^{-6}\ \mathrm{s}^{-1}, \ \bar{\gamma}_m = \gamma_m \times 10^{-6}\ \mathrm{s}^{-1}$

D. Vitali et al., Phys. Rev. A **76**, 042336 (2007))

When $\Delta \approx -\omega_m$

$$E_{\mathcal{N}} \leqslant \ln\left(\frac{1 + G/\sqrt{2\kappa\gamma_m}}{1+\bar{n}}\right)$$

Limited by the stability condition $G < \sqrt{2\kappa\gamma_m}$ and extremely fragile wrt temperature

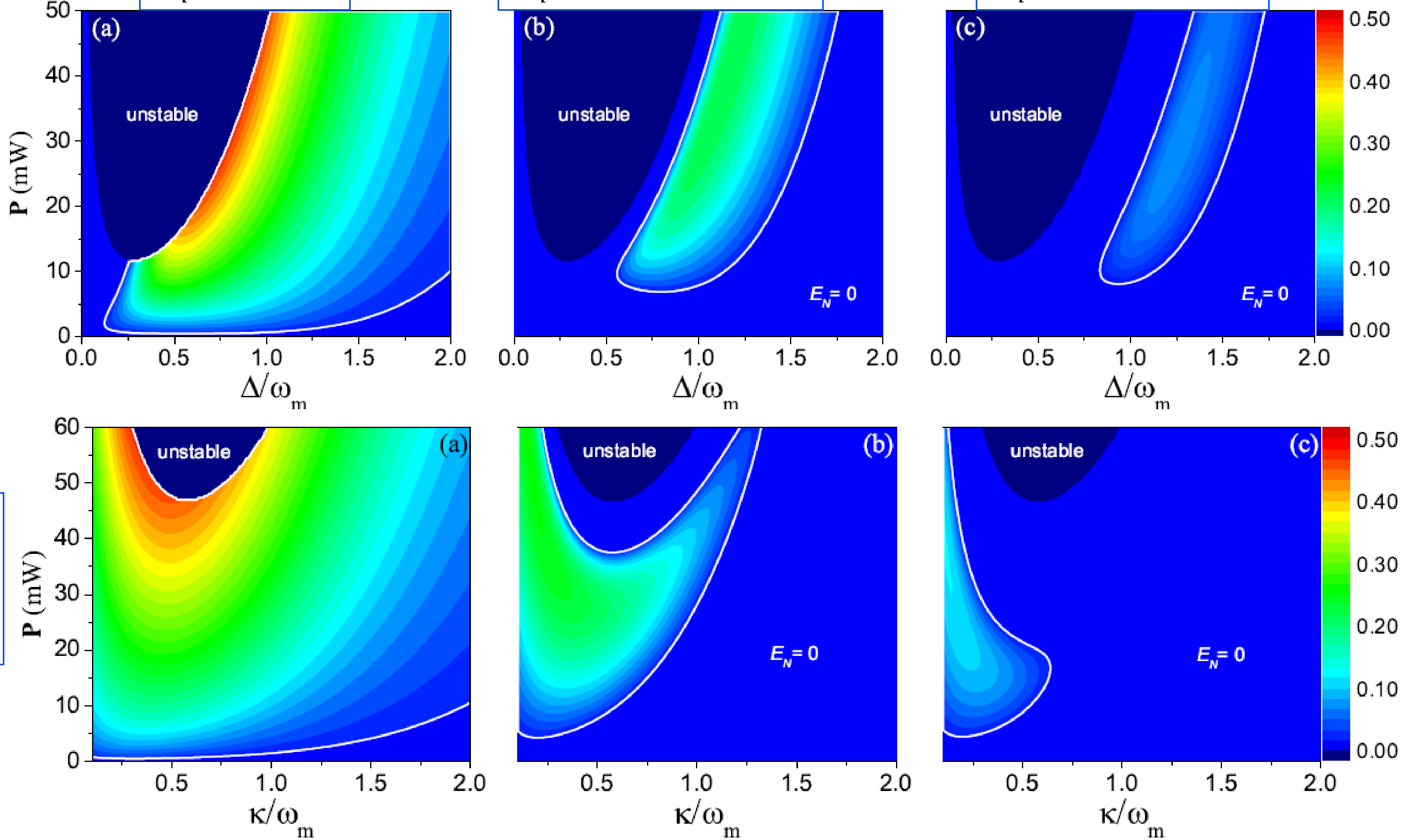
EFFECT OF PHASE NOISE ON log-neg E_N

(no phase noise)

$\Gamma_l/2\pi = 0$ | $\Gamma_l/2\pi = 0.1$ kHz | $\Gamma_l/2\pi = 1$ kHz

$2\kappa = \omega_m$

$\Delta_l = \omega_m$



State-of-art experimental parameters: $m=10$ ng, $\omega_m/2\pi = 10$ MHz, $\gamma_m/2\pi = 5$ Hz, $G_0 = 1$ kHz, $L = 1$ mm, $T = 400$ mK, $\Omega/2\pi = 50$ kHz, $\gamma = \Omega/2$

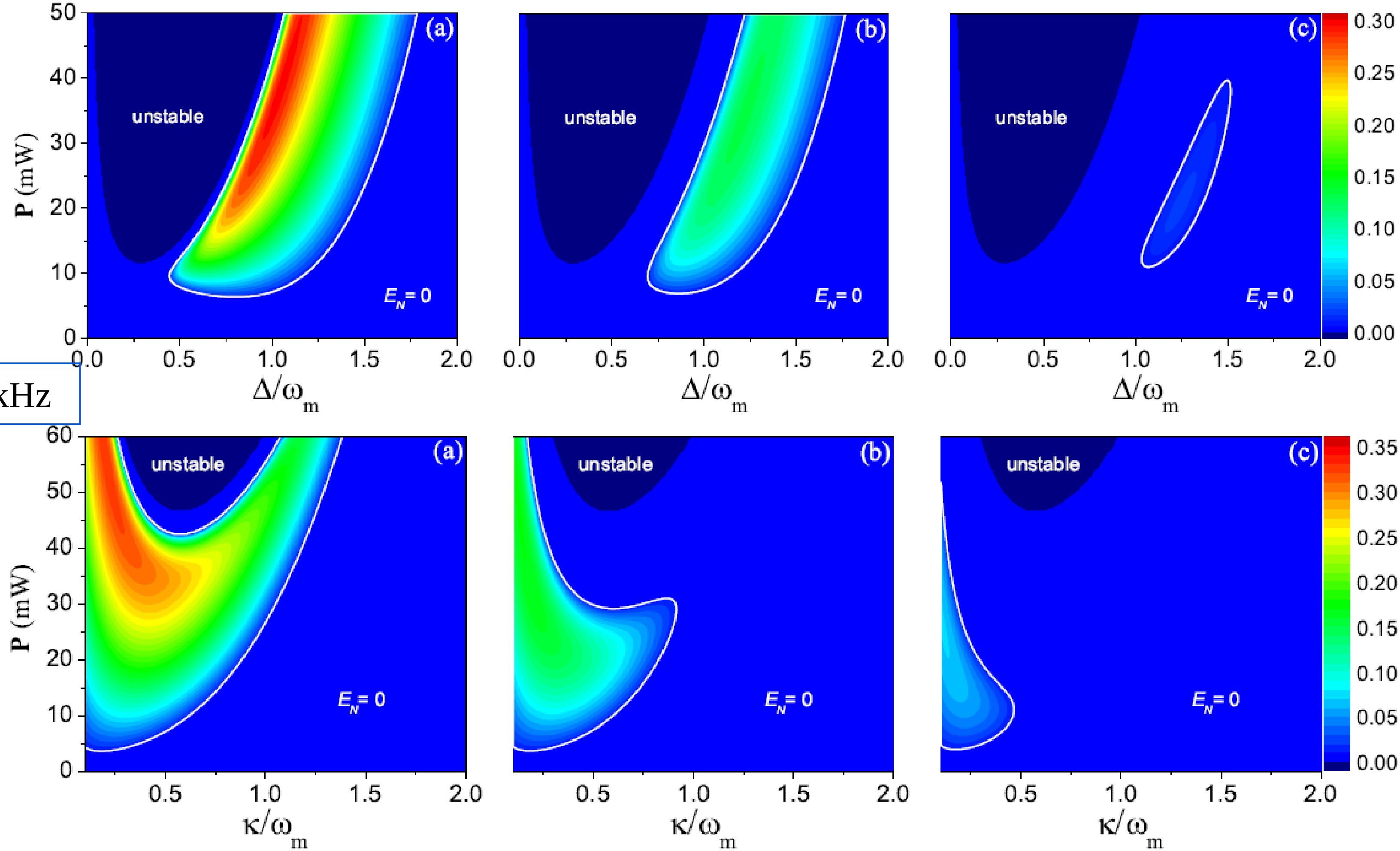
EFFECT OF PHASE NOISE **SPECTRUM** ON log-neg E_N

$\Omega/2\pi$ = 30 kHz | $\Omega/2\pi$ = 80 kHz | $\Omega/2\pi$ = 140 kHz

$2\kappa = \omega_m$

$\Gamma_l/2\pi$ = 0.1 kHz

$\Delta_l = \omega_m$



(the bandwidth parameter $\tilde{\gamma}$ is correspondingly adjusted so that $\tilde{\gamma} = \Omega/2$)

M. Abdi et al., PRA 84, 032325 (2011)

APPROXIMATE ANALYTICAL RESULTS

Relevant quantity: $\boxed{\mathcal{S}_{\dot\phi}(\omega_{\mathrm{m}}^{\mathrm{eff}})}$ Frequency noise spectrum at the **effective mechanical resonance $\omega_m{}^{eff}$**

$$\boxed{\omega_m^{eff} \approx \sqrt{\omega_m^2 - \frac{G^2 \Delta\omega_m \left[\kappa^2 - \omega_m^2 + \Delta^2\right]}{\left[\kappa^2 + (\omega_m + \Delta)^2\right]\left[\kappa^2 + (\omega_m - \Delta)^2\right]}}}$$

Frequency modified by the optomechanical coupling

Without phase noise: **E_N is maximum at the bistability threshold: close to it one has**

$$\boxed{E_N = \max(0, -\ln 2\eta^-)}$$

$$\boxed{\eta^- \simeq \frac{1}{\sqrt{2}}\sqrt{\frac{a + b\mathcal{S}_{\dot\phi}(\omega_{\mathrm{eff}}) + c\mathcal{S}_{\dot\phi}(\omega_{\mathrm{eff}})^2 + d\mathcal{S}_{\dot\phi}(\omega_{\mathrm{eff}})^3}{f + g\mathcal{S}_{\dot\phi}(\omega_{\mathrm{eff}})}}}$$

When $\mathcal{S}_{\dot\phi}(\omega_{\mathrm{eff}}) = 0$

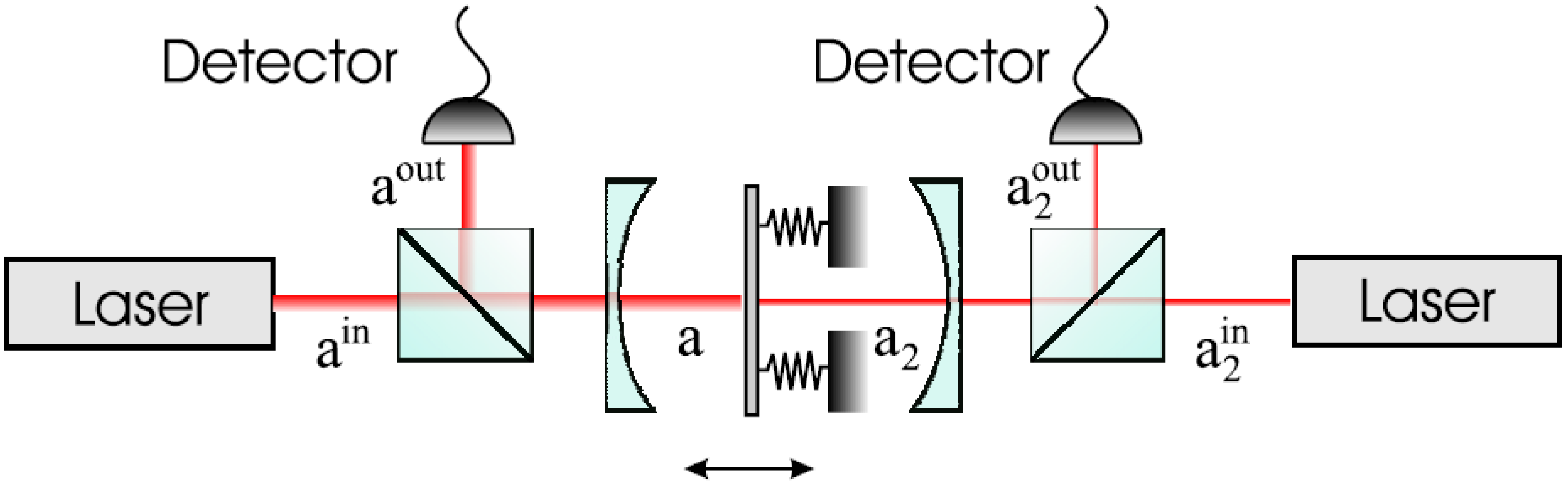
$$\boxed{\eta^- \simeq \sqrt{\frac{a}{2f}} = \sqrt{\frac{4\Delta^4 + 4\Delta^2(\kappa^2 + \omega_{\mathrm{m}}^2) + \omega_{\mathrm{m}}^4}{16\Delta^2(\Delta^2 + \kappa^2 + 5\omega_{\mathrm{m}}^2)}}}$$

Maximizing over $\Delta \Rightarrow$ $\boxed{E_N = -\ln\left[\frac{1}{5}\sqrt{9 + \frac{128\kappa^2}{8\kappa^2 + 45\omega_{\mathrm{m}}^2}}\right]}$ $\boxed{\leq \ln(5/3) \sim 0.51}$

(maximum achievable $\boldsymbol{E_N}$)

With phase noise, one easily has $\eta^- > 0.5 \Rightarrow E_N = 0$ close to bistability, and E_N becomes maximum FAR FROM threshold

DETECTION OF THE STEADY STATE



Second adjacent cavity to detect the resonator motion: if $|\alpha_2| << |\alpha_s|$ and $G_2|\alpha_2| << \kappa_2$

$$\tilde{a}_2^{out} = i\frac{G_2\alpha_2}{\sqrt{\kappa_2}}\delta\tilde{b} + \tilde{a}_2^{in}(t)$$

δb = boson operator of the mechanical mode $\Rightarrow$ **the full CM of the steady state can be reconstructed from the two output light fields**