Synthetic gauge field and synthetic dimension in interacting ultracold Fermi gases

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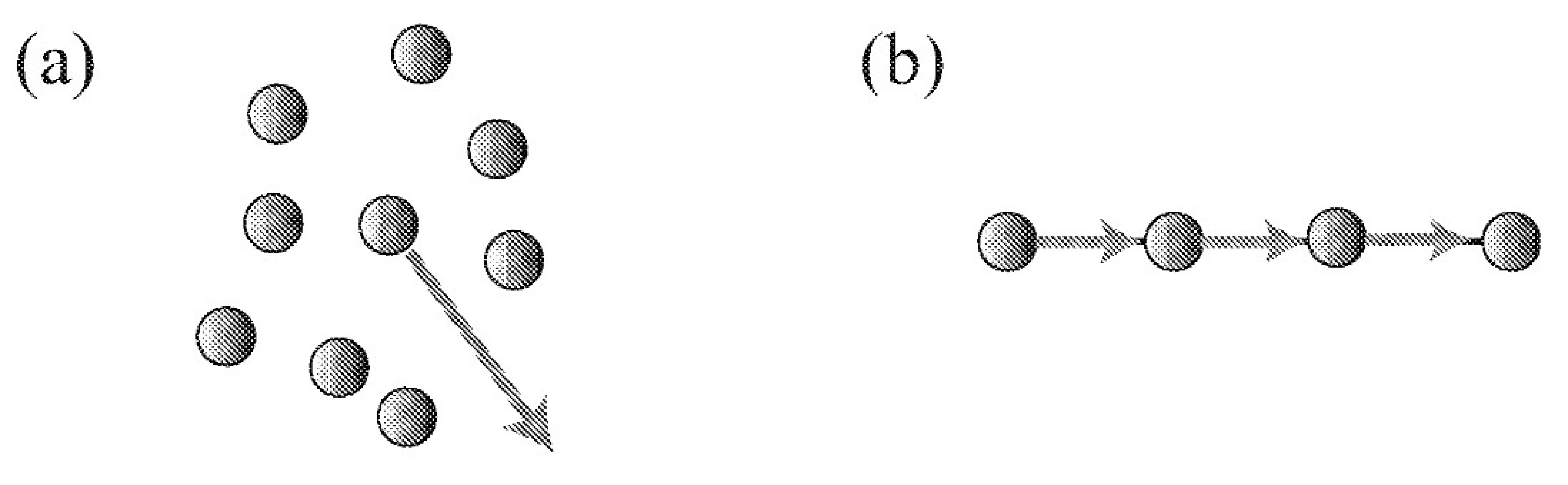
Physics in 1D

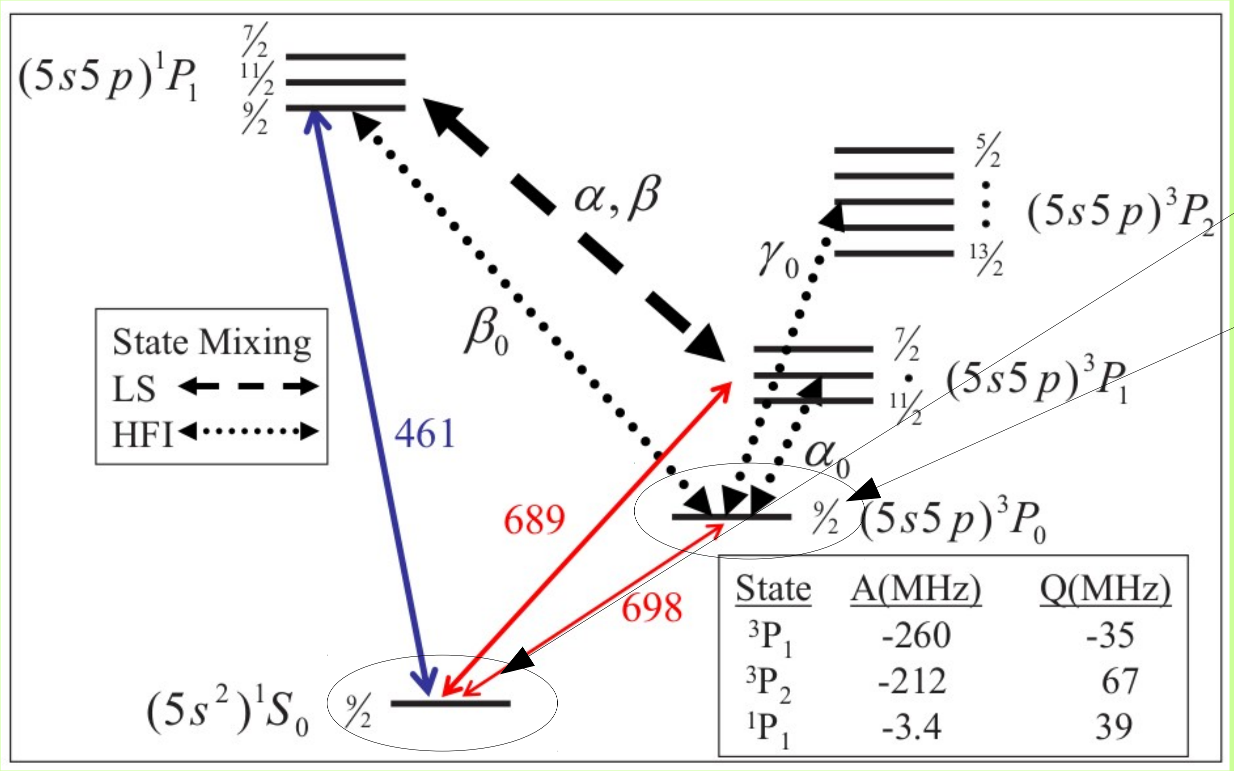
- **Interactions** are intrinsically strong → *collective* excitations
- **Statistics**: bosons and fermions behave in a similar way



Peculiar phenomena:

- Long-range order → *quasi*-long-range order (power laws)
- Solvability/integrability
- Lack of thermalization
- "Fermionization" of bosons / "bosonization" of fermions
- Anderson/many-body localization
- Spin-charge separation
- ...

Earth-alkaline(-like) Fermi atoms



^{87}Sr, from Boyd *et al.*, PRA **76**, 022510 (2007)

1) $^1S_0$ and $^3P_0$ coupled through an ultranarrow forbidden transition

2) Purely ***nuclear spin*** → s-wave scattering lengths don't depend on it

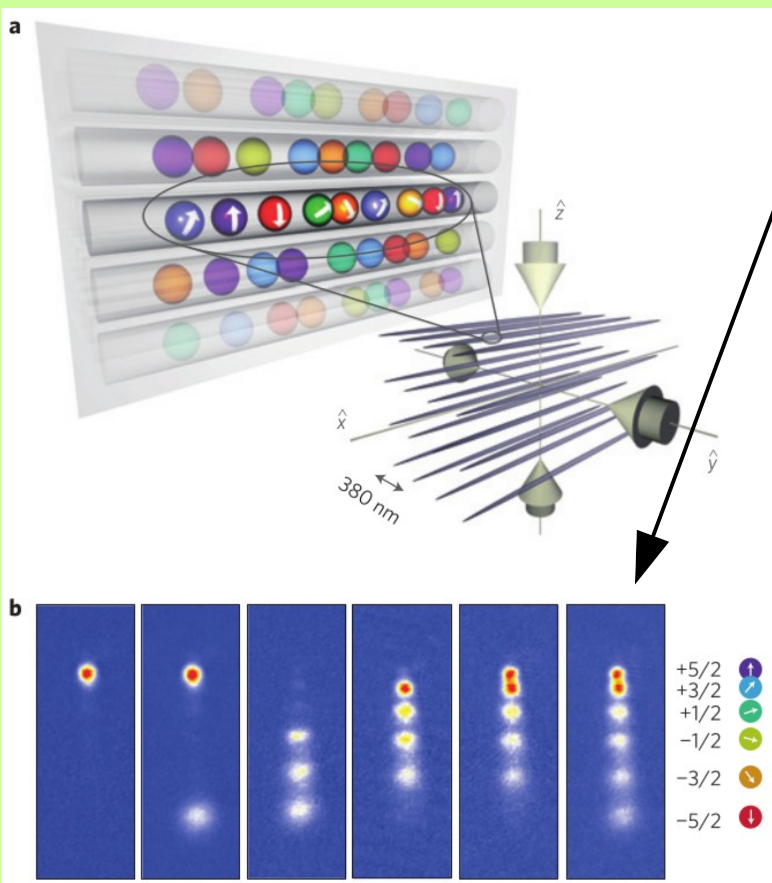
Promising platform to study physics of multi-species gases!

A first experiment in the continuum

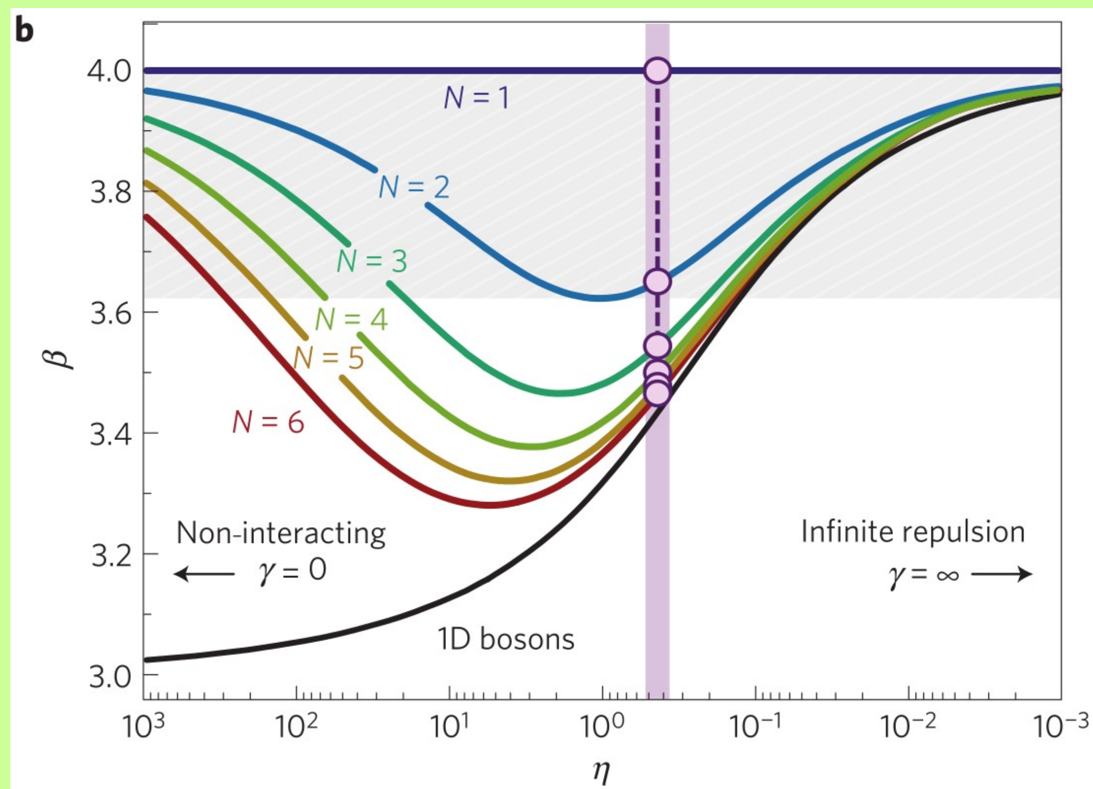
Pagano *et al.*, Nat. Phys. 10, 198 (2014)

^{173}Yb: *I = 5/2*

- The population of the $^3P_0$ orbital can be neglected
- Species (nuclear-spin m_z) can be populated at will



"Bosonization" of fermions!



Addition of the optical lattice

Gorshkov *et al.*, Nat. Phys. **6**, 289 (2010)

$$H=-\sum_{\alpha=g,e} t_\alpha \sum_j \sum_{m=-I}^{I} \left(c^\dagger_{\alpha,j,m} c_{\alpha,j+1,m} + \mathrm{H.c.}\right) + \sum_{\alpha=g,e} \frac{U_{\alpha\alpha}}{2} \sum_j n_{\alpha,j}\left(n_{\alpha,j}-1\right) + V\sum_j n_{g,j} n_{e,j} + V_{ex} \sum_{j,m,m'} c^\dagger_{g,j,m} c^\dagger_{e,j,m'} c_{g,j,m'} c_{e,j,m}$$

hopping amplitudes

on-site interaction

direct interaction

exchange interaction

Experimental realization (**Taie *et al.*, Nat. Phys. 8, 825 (2012)**), [173]Yb):

SU(N) Hubbard model

$$H_0=-t\sum_j \sum_{m=-I}^{I} \left(c^\dagger_{j,m} c_{j+1,m} + \mathrm{H.c.}\right) + U \sum_j \sum_{m<m'} n_{j,m} n_{j,m'}$$

Assaraf *et al.*, PRB **60**, 2299 (1999)
Szirmai & Sólyom, PRB **71**, 205108 (2005)
Buchta *et al.*, PRB **75**, 155108 (2007)
Manmana *et al.*, PRA **84**, 043601 (2011)
...

Observation of SU(6) Mott insulator!

Synthetic dimension

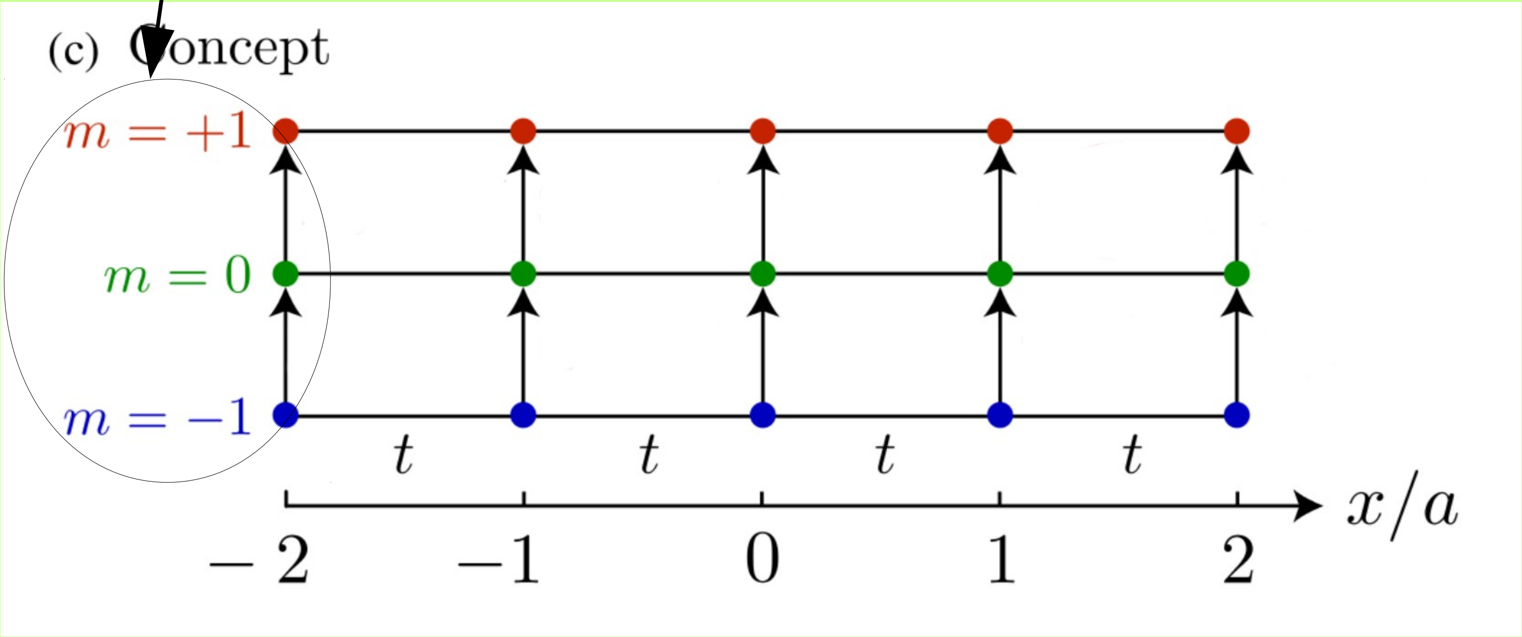
Boada *et al.*, PRL **108**, 133001 (2012)

Idea: use an internal degree of freedom of the atom (in our case, nuclear spin) as an additional dimension

Raman coupling of nuclear-spin levels → tunneling in the synthetic dimension

Quantum ladder!

quasi-1D system→ exploration of physics in **2D**

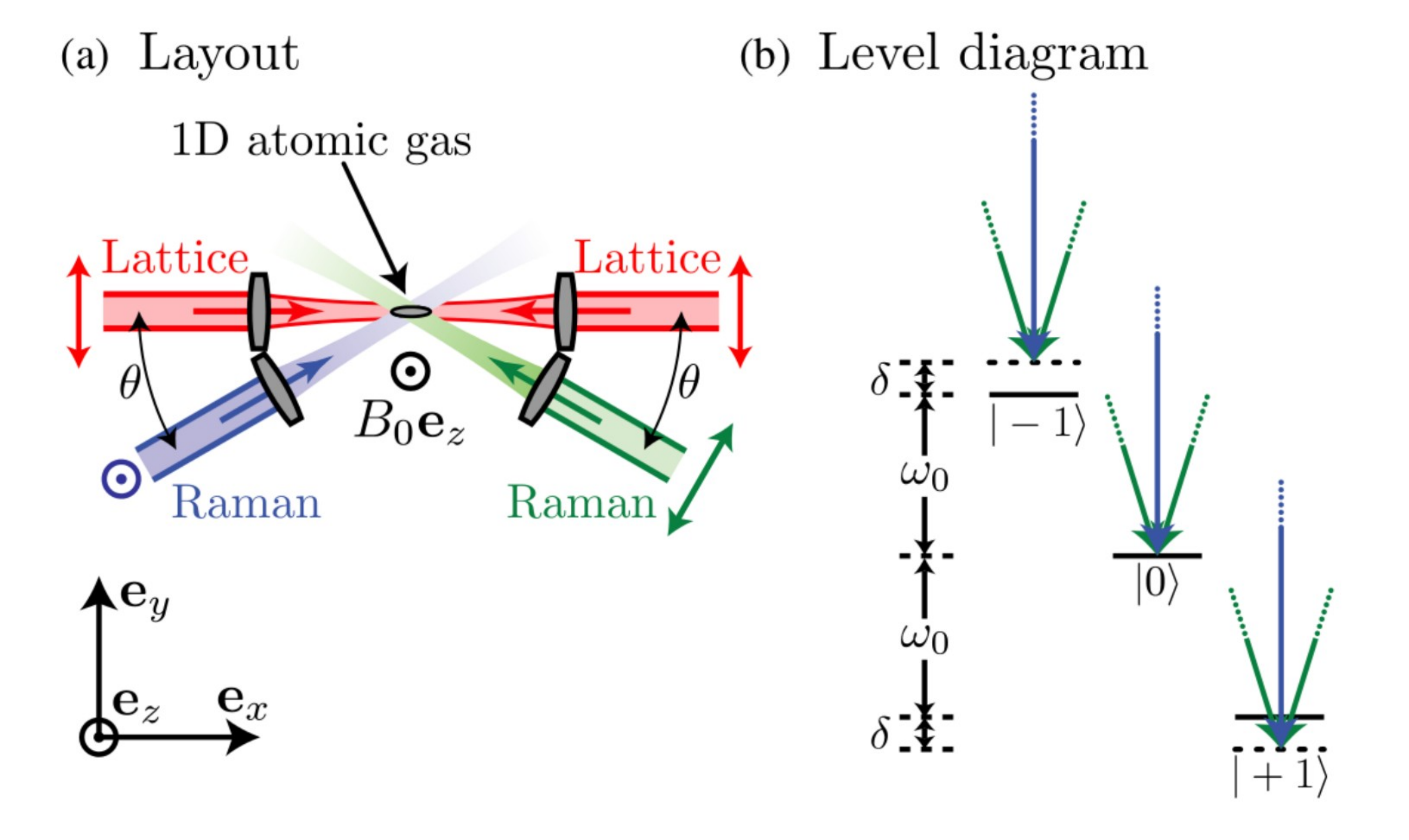


Synthetic gauge field (I)

Celi *et al.*, PRL **112**, 043001 (2014)

Question: is it possible to engineer a **magnetic-like flux** in the synthetic lattice?

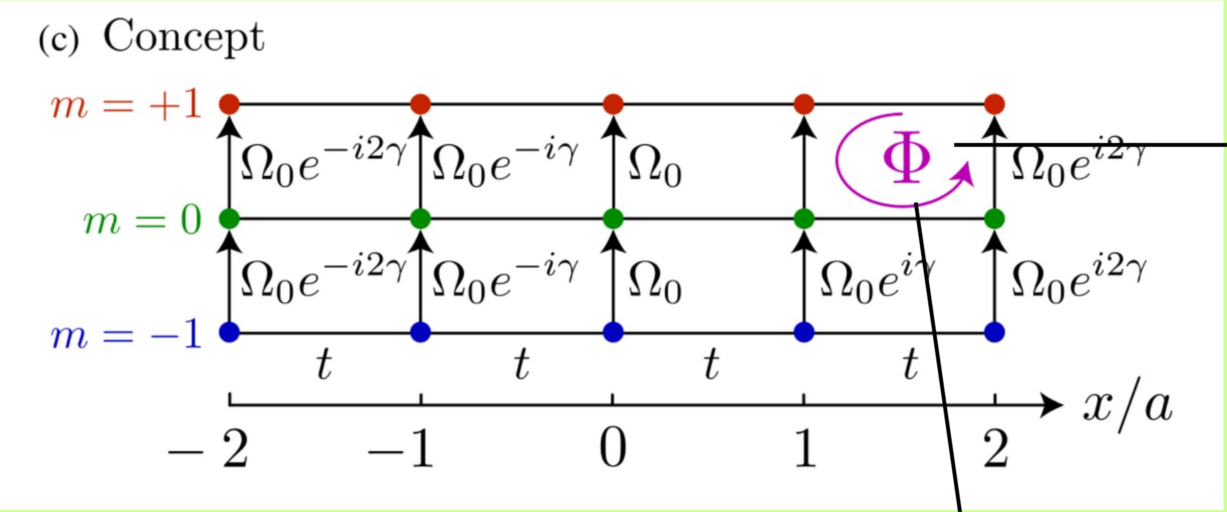
Answer: yes! Raman-assisted coupling in the synthetic dimension



(For other ways of implementing synthetic gauge fields, see, e.g., Dalibard *et al.*, RMP **83**, 1523 (2011))

Synthetic gauge field (II)

Celi *et al.*, PRL **112**, 043001 (2014)



Magnetic-like flux

$$\boxed{\Phi=\gamma}$$

$$\boxed{H=H_0+H_1+H_2}$$

(a **cyclic** coupling between $m=\pm I$ can in principle also be implemented)

$$H_0=-t\sum_j\sum_{m=-I}^{I}\left(c^\dagger_{j,m}c_{j+1,m}+\mathrm{H.\,c.}\right)+U\sum_j\sum_{m<m'}n_{j,m}n_{j,m'}$$

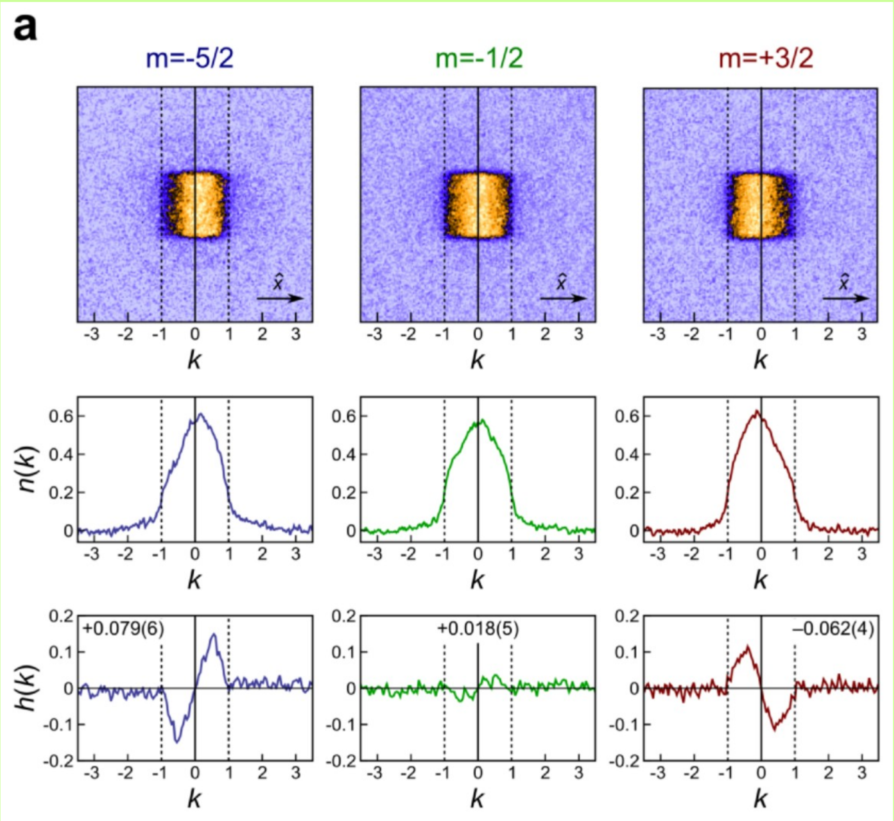
$$H_1=\Omega\sum_j\sum_{m=-I}^{I-1}\left(e^{-i\gamma j}c^\dagger_{j,m}c_{j,m+1}+\mathrm{H.c.}\right)$$

$$H_2=\Omega'\sum_j\left(e^{-i\gamma j}c^\dagger_{j,I}c_{j,-I}+\mathrm{H.c.}\right)$$

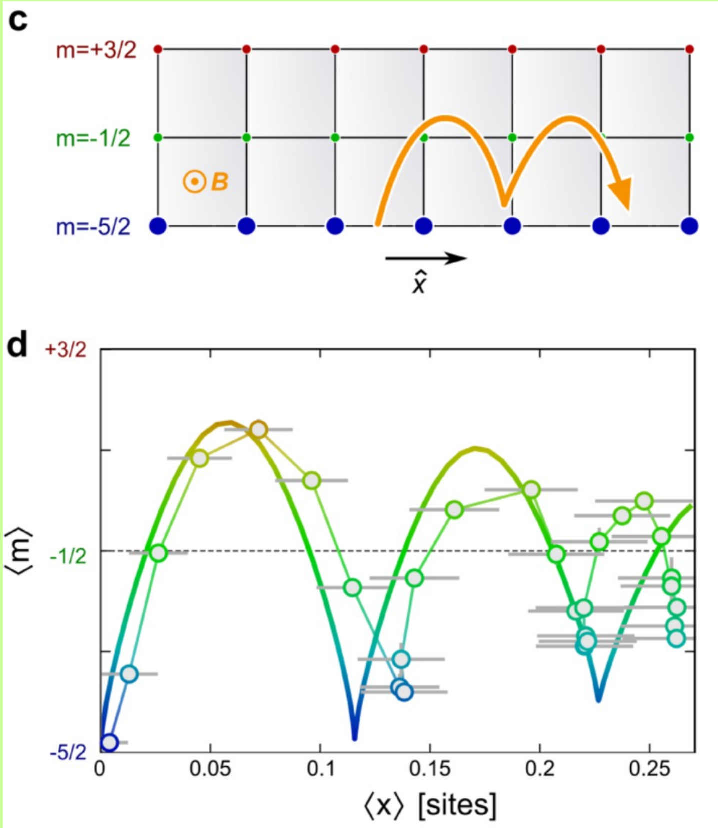
related to the parameters of Raman lasers

A look to experiments

Mancini *et al.*, Science 349, 1510 (2015)



Momentum distributions



Skipping orbits

Evidence for chiral edge modes!

Quantum Hall effect

System: 2D electrons in a magnetic field

Observation: at strong magnetic fields, Hall conductance is quantized

$$\sigma_H = \frac{p}{q}\frac{e^2}{h}$$

q = 1: *integer* QHE; q > 1: *fractional* QHE

Key quantity: *filling fraction*

number of electrons

$$\boxed{\nu = \frac{N}{N_\Phi} = \frac{p}{q}}$$

number of elementary fluxes

Microscopically: the bulk is an *insulator*, and all the conductance is concentrated on the edges

$$\downarrow$$

Edge modes!

Integer QHE: *non-interacting physics*

Fractional QHE: Coulomb **interaction**

$$\downarrow$$

Liquid: Laughlin wave functions, at $\nu = \frac{1}{q}$ q odd

See, for a review, Prange & Girvin, *The Quantum Hall Effect* (1990)

Integer QHE with cold atoms

$$H_0 = -t\sum_j \sum_{m=-I}^{I} \left(c^\dagger_{j,m} c_{j+1,m} + \text{H. c.}\right) + \cancel{U \sum_j \sum_{m<m'} n_{j,m} n_{j,m'}}$$

$$H_1 = \Omega \sum_j \sum_{m=-I}^{I-1} \left(e^{-i\gamma j} c^\dagger_{j,m} c_{j,m+1} + \text{H.c.}\right)$$

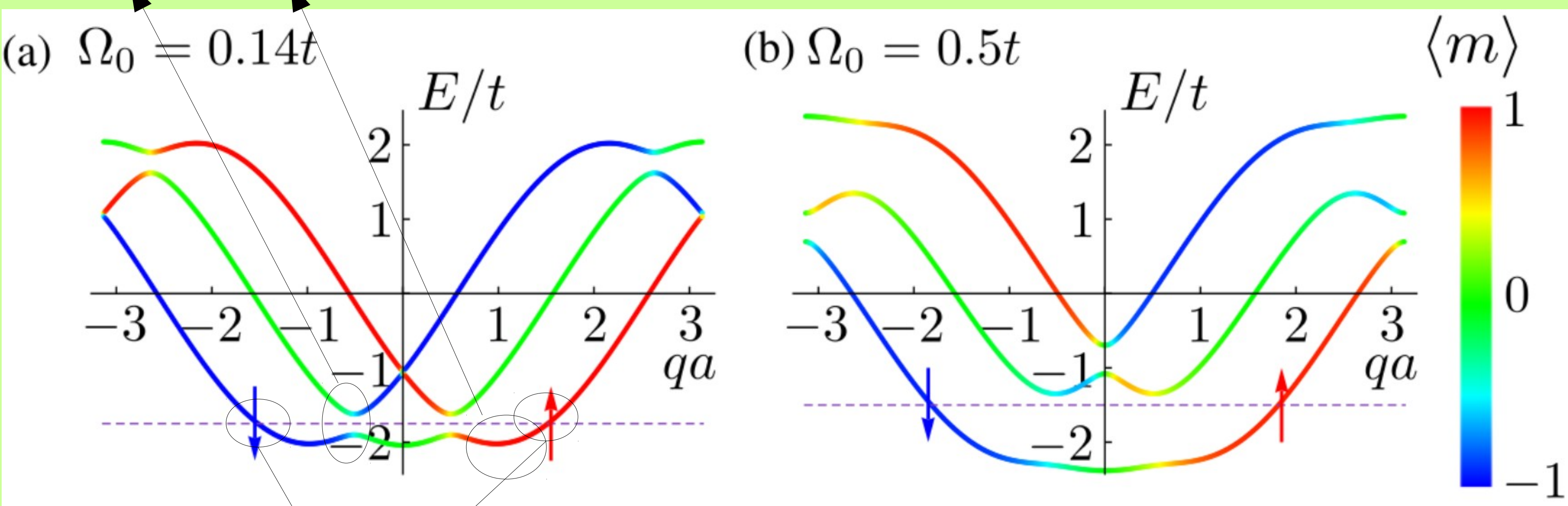
$$\cancel{H_2 = \Omega' \sum_j \left(e^{-i\gamma j} c^\dagger_{j,I} c_{j,-I} + \text{H.c.}\right)}$$

Filling fraction:

$$\boxed{\nu = \frac{N}{L(2I+1)\frac{\gamma}{2\pi}} \in \mathbb{N}}$$

$\boxed{\sim\Omega}$ $\boxed{\gamma}$

Single-particle spectrum:



From Celi *et al.*, PRL **112**, 043001 (2014)

Helical modes: **gapless** low-energy excitations with definite **momentum** and **spin**!

►Edge modes of QHE

Questions

#0 To what extent the system is a quantum simulator of QHE?

#1 What happens when interactions are added?

#2 What happens when the cyclic coupling in the synthetic direction is added?

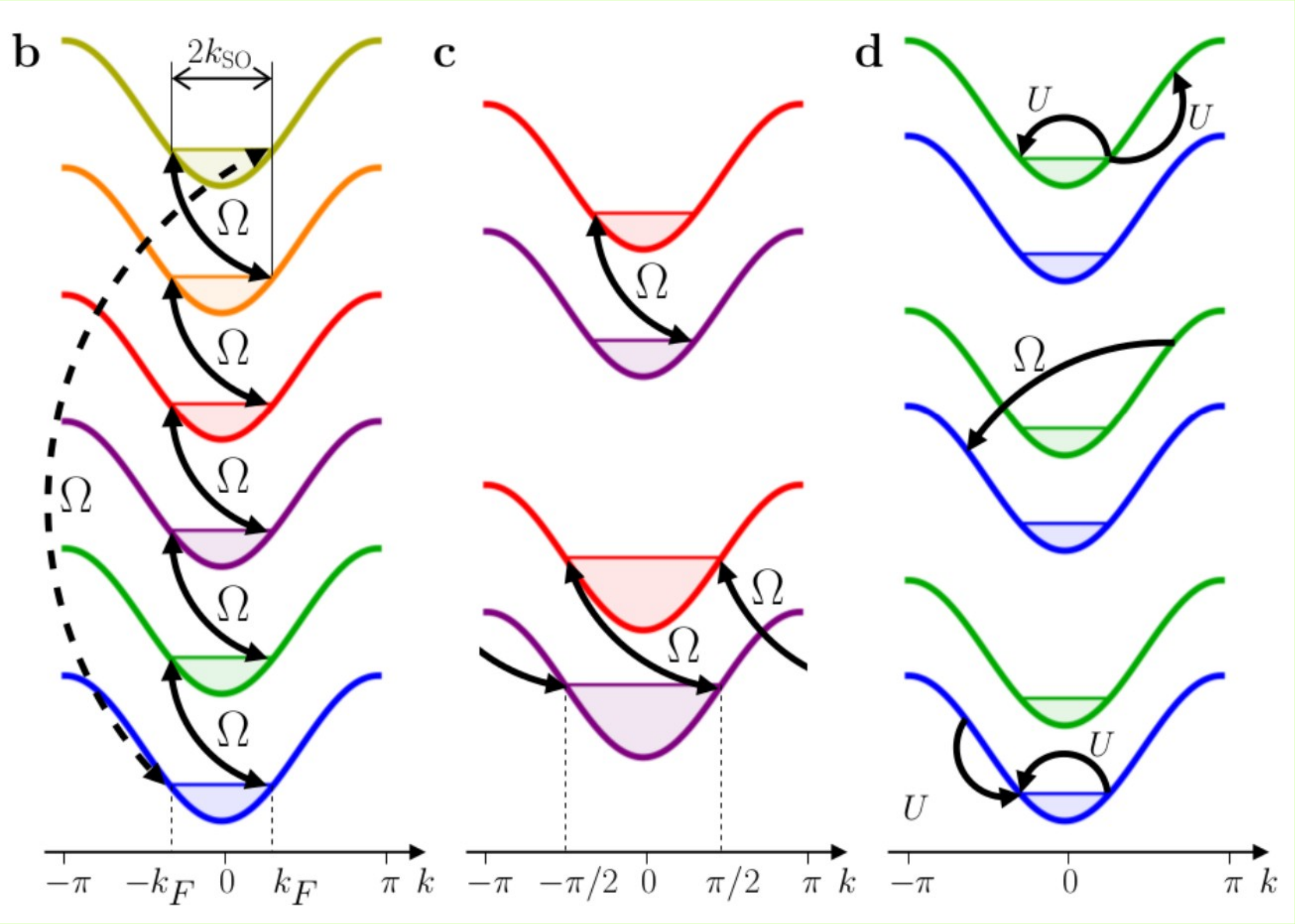
#3 Which observables can characterize the properties of the system?

Cyclic coupling

$$H_0 = -t\sum_j \sum_{m=-I}^{I} (c^\dagger_{j,m} c_{j+1,m} + \text{H. c.}) + U \sum_j \sum_{m<m'} n_{j,m} n_{j,m'}$$

$$H_1 = \Omega \sum_j \sum_{m=-I}^{I-1} (e^{-i\gamma j} c^\dagger_{j,m} c_{j,m+1} + \text{H.c.})$$

$$\Omega \leftarrow H_2 = \Omega' \sum_j (e^{-i\gamma j} c^\dagger_{j,I} c_{j,-I} + \text{H.c.})$$



Barbarino, LT *et al.*, Nat. Comm. **6**, 8134 (2015)

Each term in H_1 opens a gap $\sim\Omega$ in the single particle spectrum → in the non-interacting case, the system becomes fully gapped at fillings $\nu \in \mathbb{N}$ → $N(\nu, \gamma)$

↓

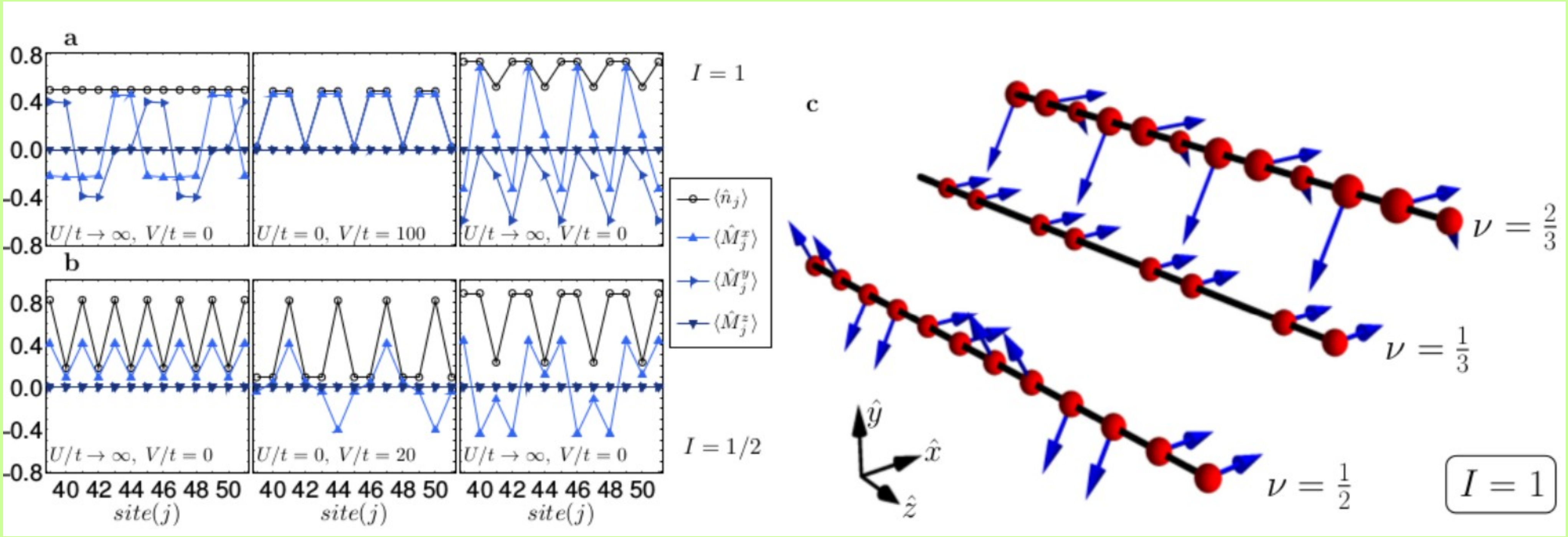
When $\Omega, U \neq 0$, a many-body gap opens at fillings $\nu \in \mathbb{Q}$

↓

Fractional QHE!

Magnetic crystals #1

Computational method: **DMRG** (White, PRL **69**, 2863 (1992))



Crystalline order!

"Problem": decreasing ν , **longer-ranged interactions** are needed in order to stabilize the crystal $\longrightarrow$ Dipoles?

Barbarino, LT *et al.*, Nat. Comm. **6**, 8134 (2015)

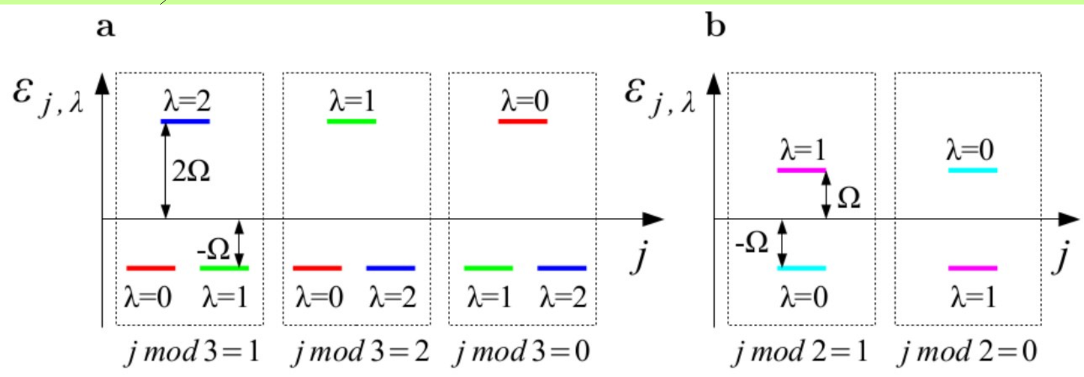
Magnetic crystals #2

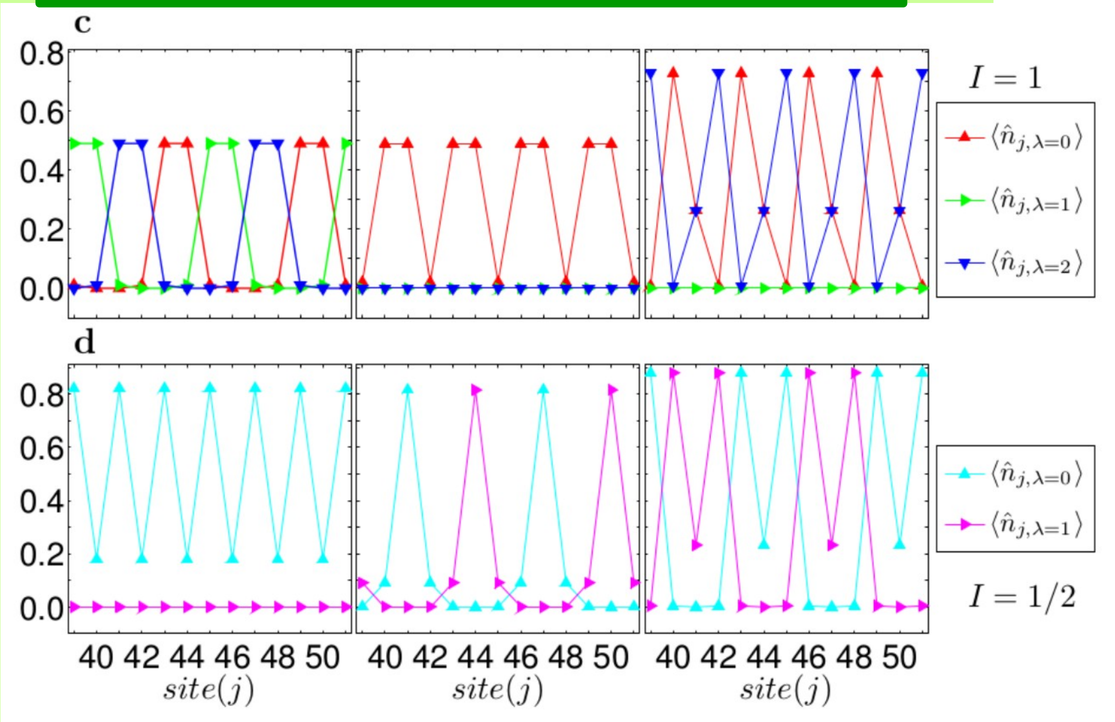
$$H_0 = -t \sum_j \sum_{m=-I}^{I} \left(c^\dagger_{j,m} c_{j+1,m} + \text{H. c.}\right) + U \sum_j \sum_{m<m'} n_{j,m} n_{j,m'}$$

$$H_1 = \Omega \sum_j \sum_{m=-I}^{I-1} \left(e^{-i\gamma j} c^\dagger_{j,m} c_{j,m+1} + \text{H.c.}\right)$$

$$H_2 = \Omega' \sum_j \left(e^{-i\gamma j} c^\dagger_{j,I} c_{j,-I} + \text{H.c.}\right)$$

True structure of the crystals!





Diagonalization (Fourier)

$$\begin{cases} d_{j,\lambda} = \dfrac{1}{\sqrt{2I+1}} \sum_m e^{i\frac{2\pi\lambda}{2I+1}m} c_{j,m} \\ \epsilon_{j,\lambda} = 2\Omega \cos\left(\dfrac{2\pi\lambda}{2I+1} + \gamma j\right) \end{cases}$$

Barbarino, LT *et al.*, Nat. Comm. **6**, 8134 (2015)

Helical liquids

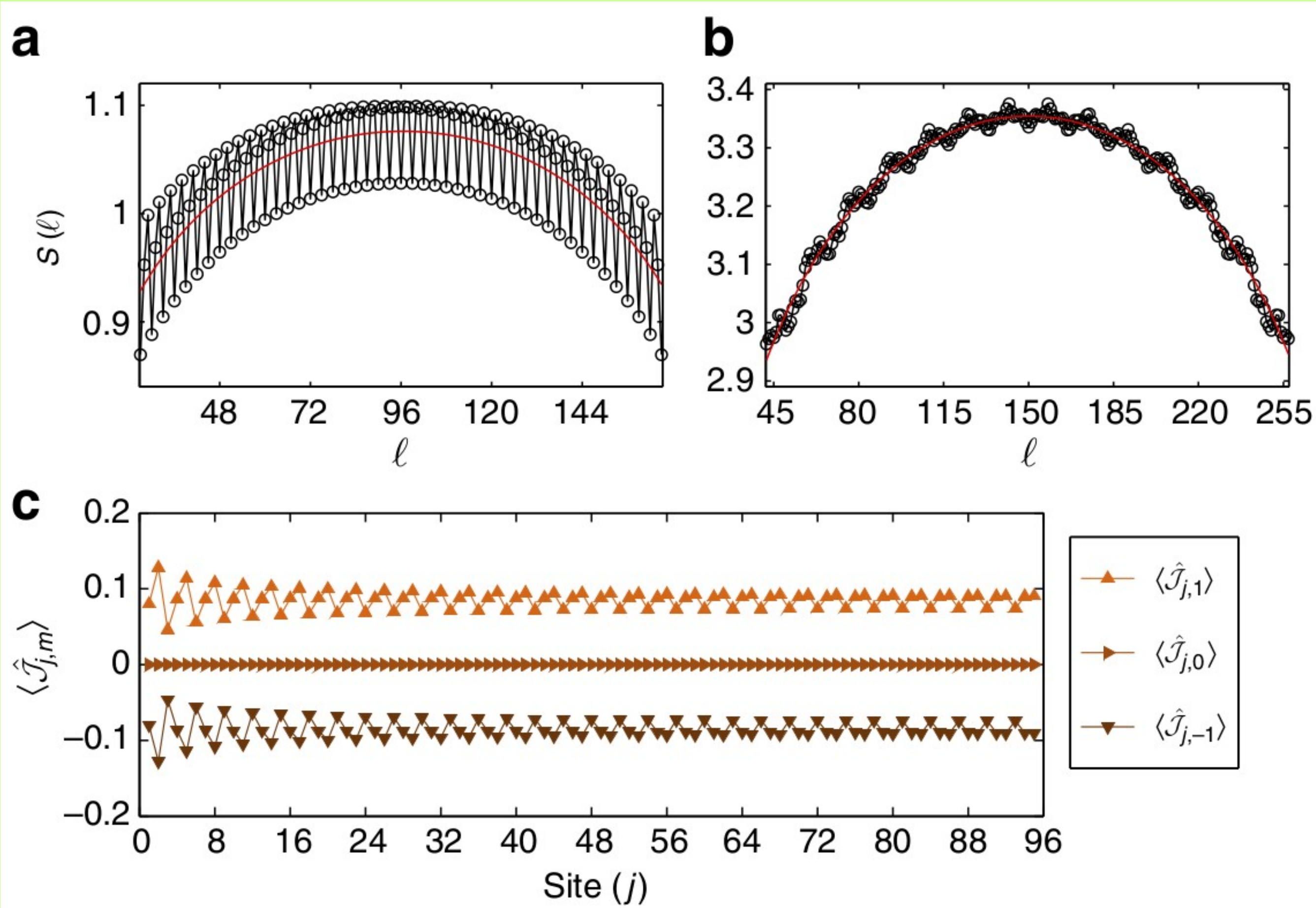
$$H_0 = -t\sum_j \sum_{m=-I}^{I} \left(c^\dagger_{j,m} c_{j+1,m} + \text{H. c.}\right) + U\sum_j \sum_{m<m'} n_{j,m} n_{j,m'}$$

$$H_1 = \Omega \sum_j \sum_{m=-I}^{I-1} \left(e^{-i\gamma j} c^\dagger_{j,m} c_{j,m+1} + \text{H.c.}\right)$$

$$\cancel{H_2 = \Omega' \sum_j \left(e^{-i\gamma j} c^\dagger_{j,I} c_{j,-I} + \text{H.c.}\right)}$$

→ Helical liquids should survive!

Spin-resolved currents + entanglement entropies → central charge



Filling, Raman coupling and interactions are all necessary in order to stabilize the liquids!

Barbarino, LT *et al.*, Nat. Comm. **6**, 8134 (2015)

Helicality and interactions

Questions:
Is helicality affected by **interactions**?
Can we find experimentally detectable quantities that witness chirality?

Momentum distribution functions

$$n_{p,m}, \quad n_p = \sum_m n_{p,m}$$

Mean current

$$Q_m = \frac{1}{L} \sum_j C_{j,m} = \frac{-2t}{L} \sum_{p>0} \left(n_{p,m} - n_{-p,m} \right)$$

Asymmetry of the momentum distributions

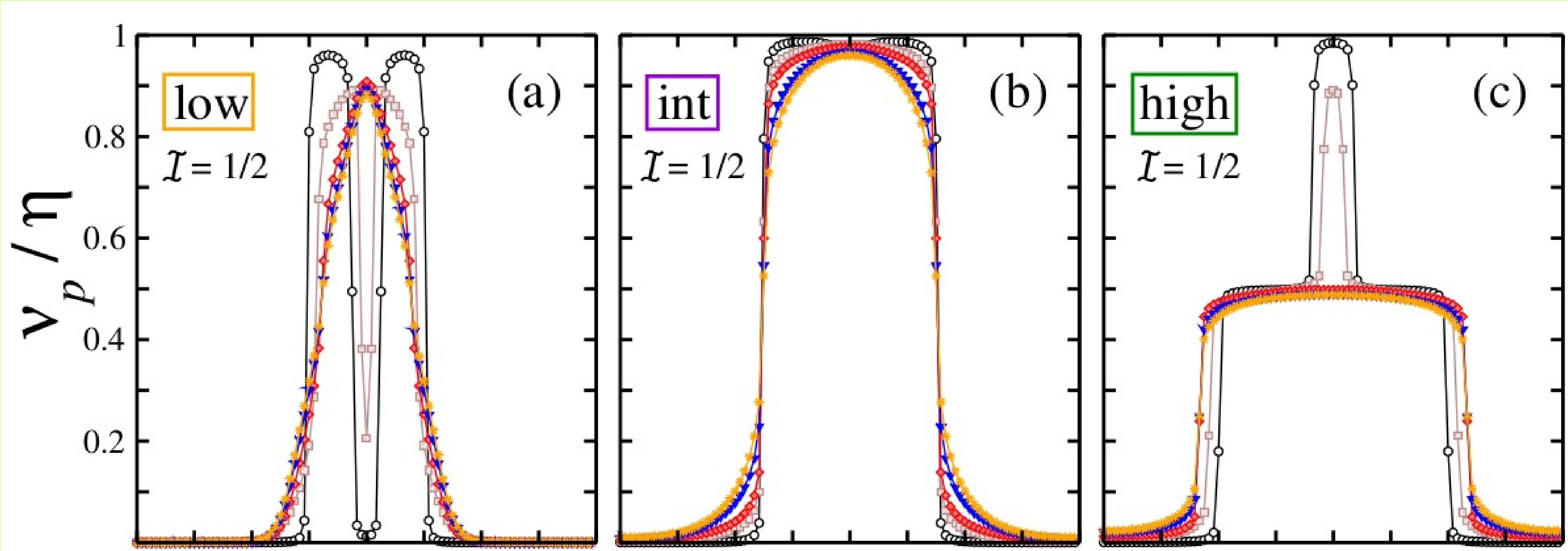
$$J_m = -\sum_{p>0} \left(n_{p,m} - n_{-p,m} \right)$$

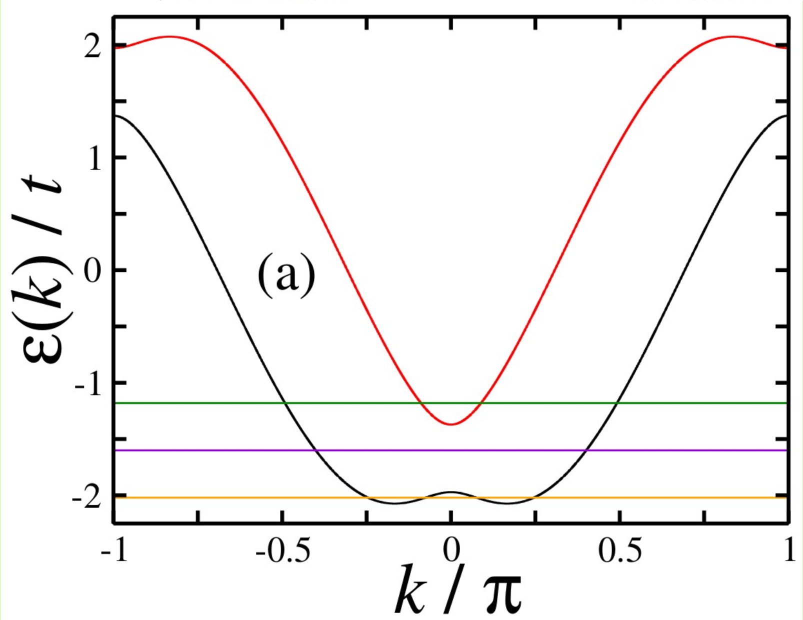
Key observables: momentum distributions!

Mancini *et al.*, Science **349**, 1510 (2015)

Barbarino, LT *et al.*, arXiv:1510.05603

Momentum distributions #1



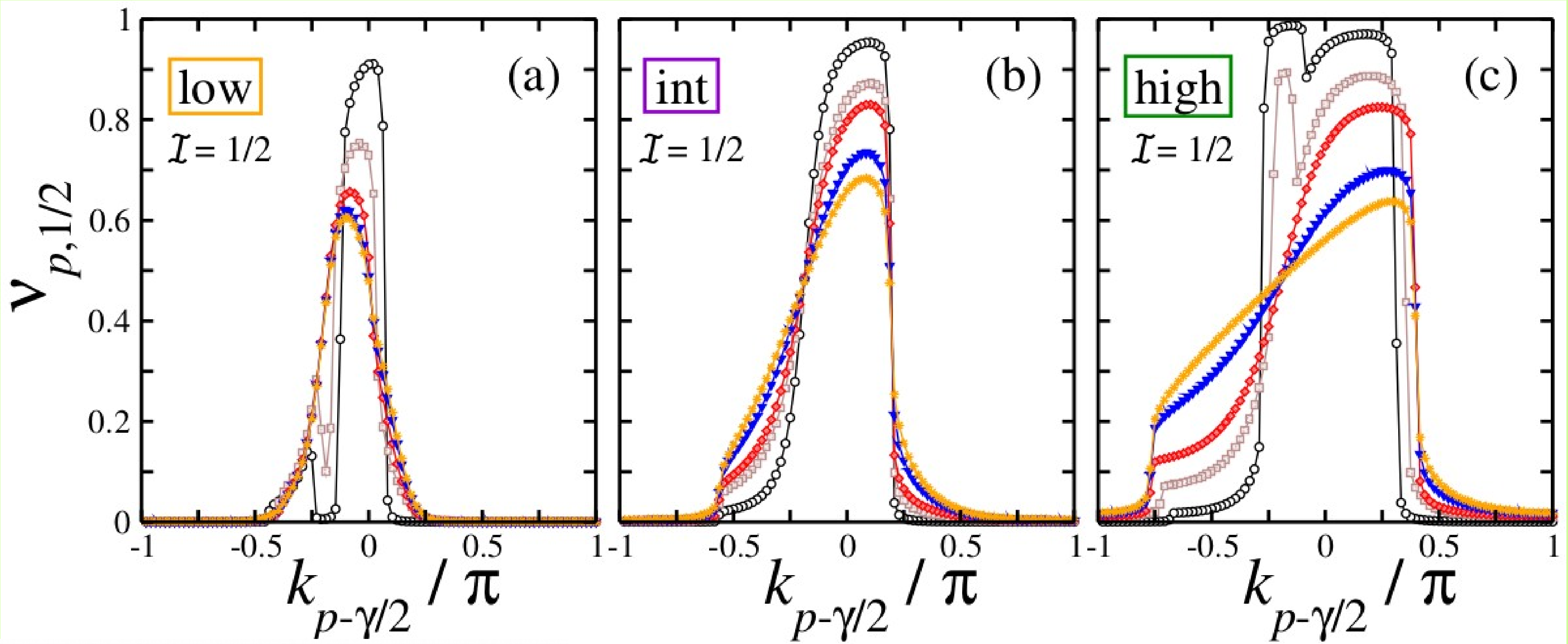


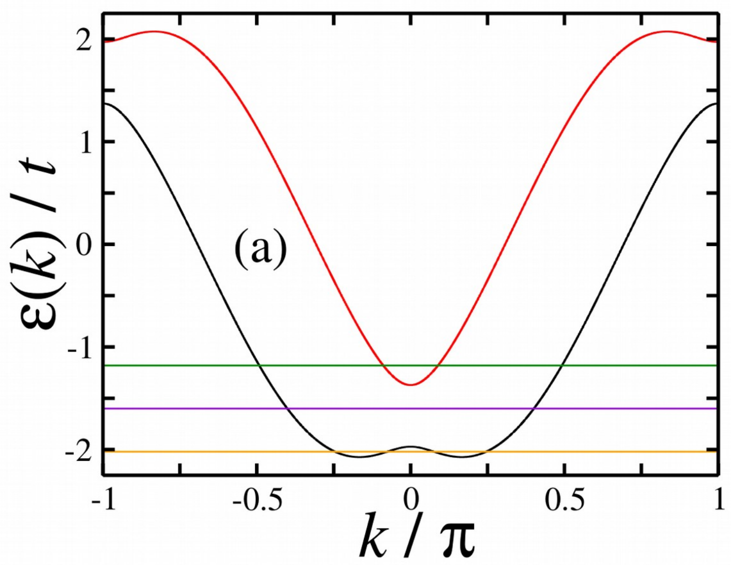
The total momentum distributions show that increasing *U* results in an **effective increasing of** Ω

Theoretical prediction from Braunecker *et al.*, PRB **82**, 045127 (2010)

Barbarino, LT *et al.*, arXiv:1510.05603

Momentum distributions #2

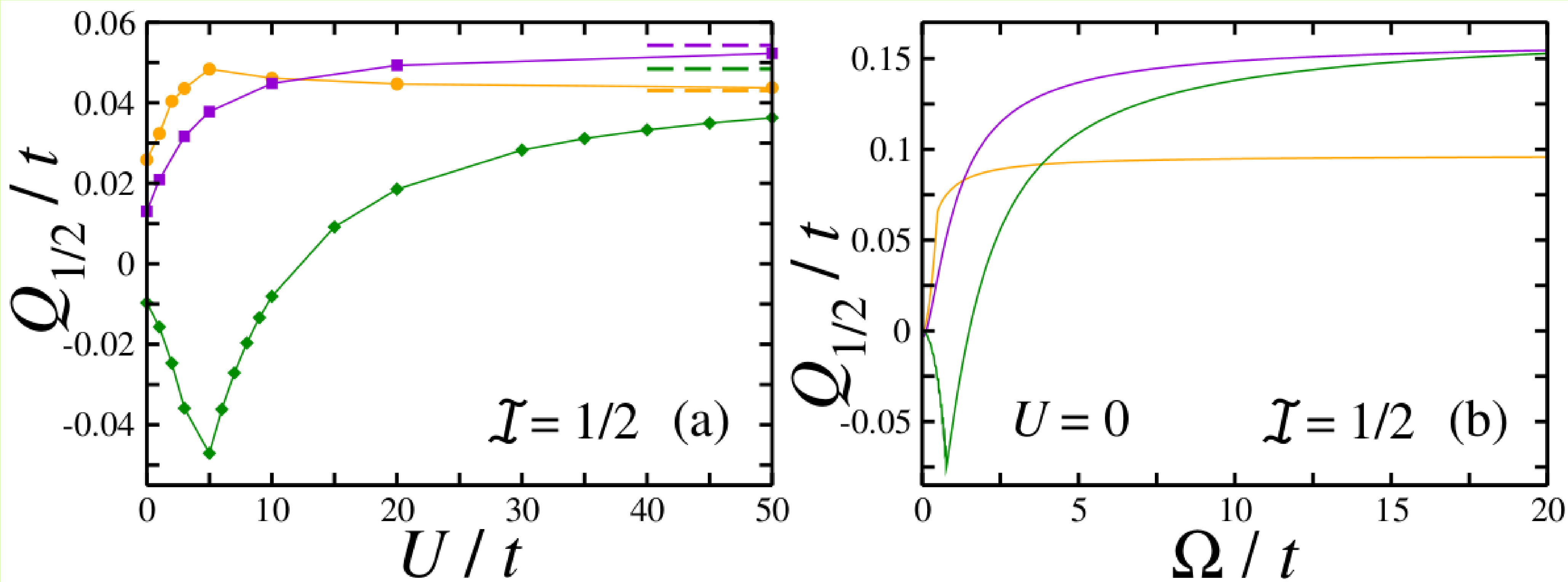


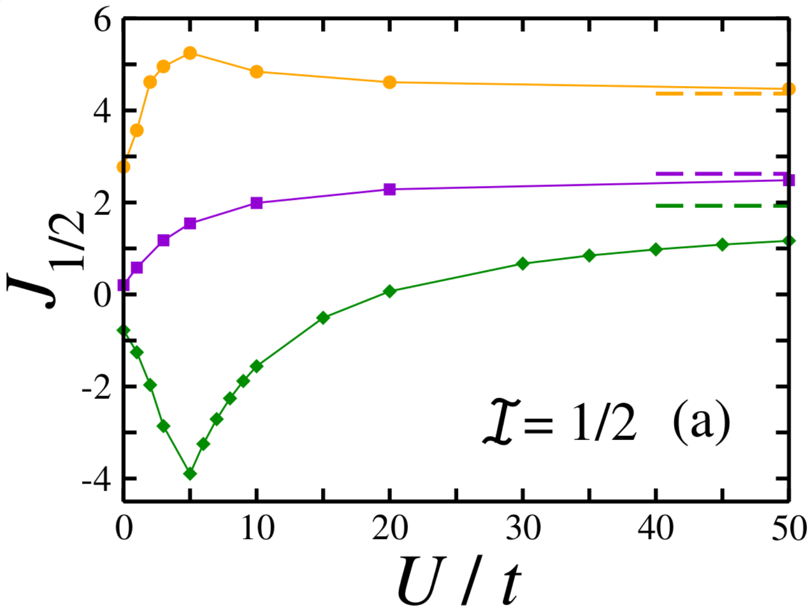


The shape of the spin-resolved momentum distributions changes increasing U → **asymmetry wrt k = 0 is enhanced**

Barbarino, LT *et al.*, arXiv:1510.05603

Asymmetry and mean current





- Mean currents display non-trivial behaviors very similar each other
- The behaviors are explained by the effective enhancement of Ω

Barbarino, LT *et al.*, arXiv:1510.05603

Conclusions and outlook

- Earth-alkaline-like atoms offer a promising platform for the study of the QHE in cold atoms
- We showed that some features of QHE emerge when a synthetic gauge field is implemented in the lattice gas
- We show how helicality witnesses are affected by interactions

Some other interesting directions:

- In which regimes is the system QHE-like? → Phase diagram
- What is the role of the form of the interaction for the QHE physics?
- Is the system topological?
- Does the system possess fractional/anionic excitations?
- ...

Thank you for your attention!