

# The physics of disorder with a Bose-Einstein condensate

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Pisa CNR-INO, 5 Dicembre 2013



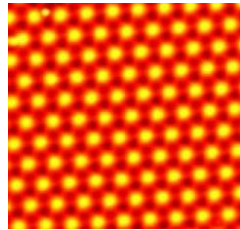
# Disorder in quantum systems

Disorder is ubiquitous in nature and rules the behavior of many physical systems.

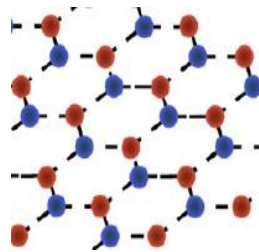
Fundamental phenomena: Anderson localization, interaction vs disorder, strongly-correlated spin glasses, etc.



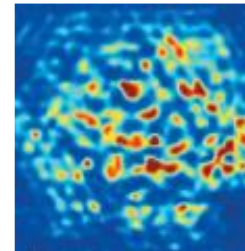
Superfluids  
in porous media



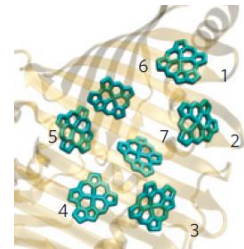
Superconductors



Graphene



Photonic media



Biological systems

Applications: 2D superconductive films, graphene, photonic media, biological systems, etc.

# Ultracold atoms

Quantum gas :

Atoms cooled down to quantum degeneracy

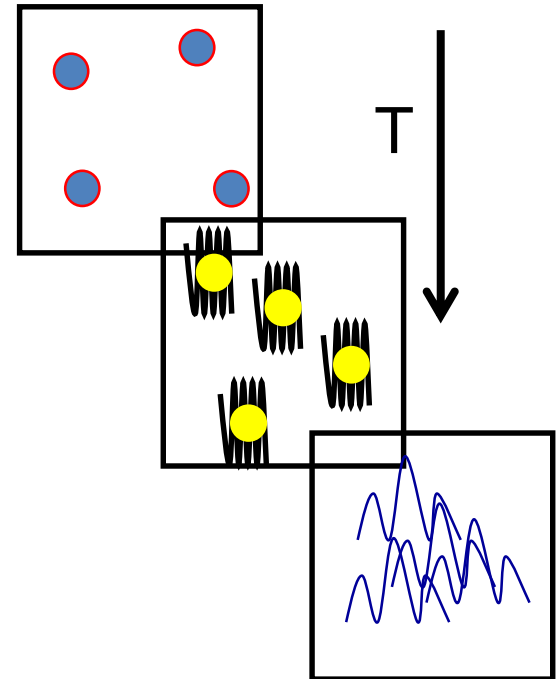
Matter wave with interaction -> superfluidity

Quantum gas experiment :

extremely versatile tool characterized by

good control and large tunability of the system parameters

- Statistics (Bosons or fermions)
- Dimensionality (1D, 2D, 3D)
- Shaping of the potential (optical potentials)
- Control and tune of the interactions (attractive/repulsive, short/long range)

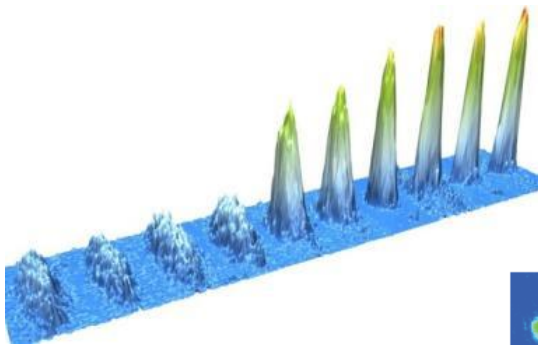


# The physics of disorder with ultracold atoms

Disorder is hard to model in theory and to control and tune in both real and experimental systems.

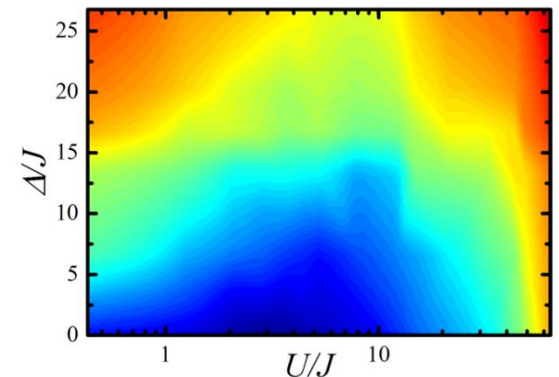
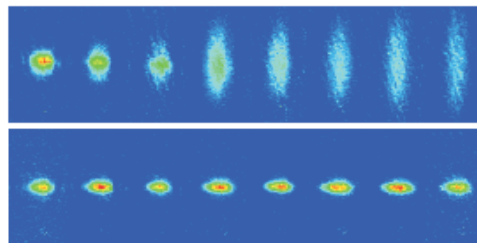
Particularly challenging is the study of the combined effect of disorder and interactions.

## Disorder in Florence!



Anderson localization of matter waves

Transport in a disordered interacting system



Glassy phases of disordered interaction bosons

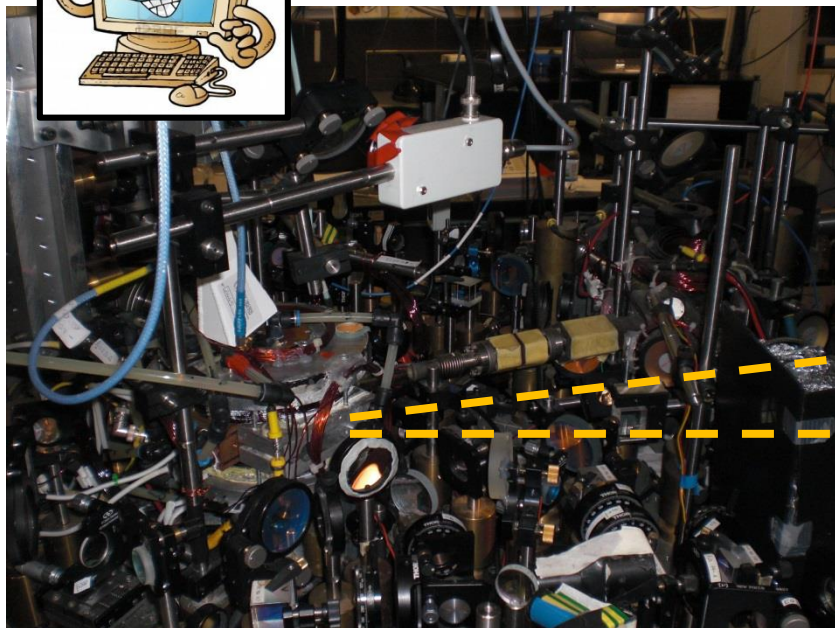
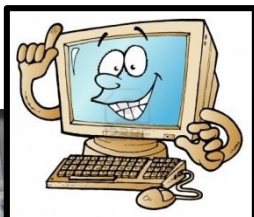
# Quantum simulations with cold atoms

Solutions of Hamiltonians in the laboratory

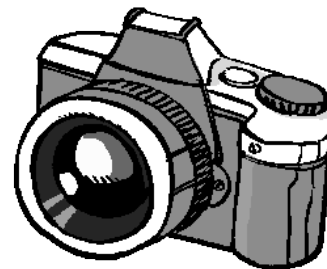
$$H\psi = E\psi$$

Ex: 1D Disordered Bose-Hubbard Hamiltonian

$$H = -J \sum_i (b_i^\dagger b_{i+1} + h.c.) + \Delta \sum_i \cos(2\pi\beta i) n_i + \frac{U}{2} \sum_i n_i(n_i - 1)$$



Direct access to the solutions  
in both real and momentum space  
and probing of the excitation spectrum

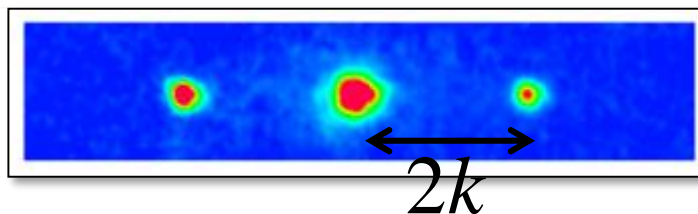
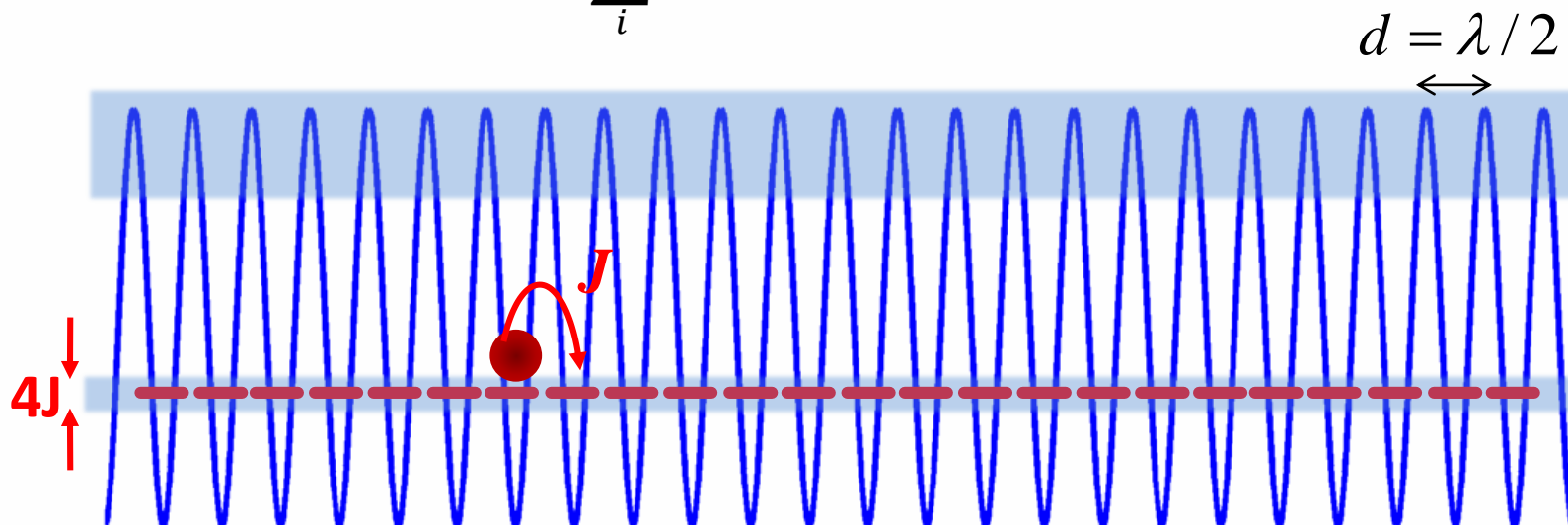


# Optical lattice

Using laser light it is possible to shape conservative potentials for neutral atoms at will, exploiting the dipole interaction between the electromagnetic field and the induced atomic dipolar momentum:

A standing wave provides a perfect periodical potential

Tight binding limit: 
$$H = -J \sum_i (b_i^\dagger b_{i+1} + h.c.)$$



$$k = \frac{2\pi}{\lambda}$$

# Disordered Optical lattice

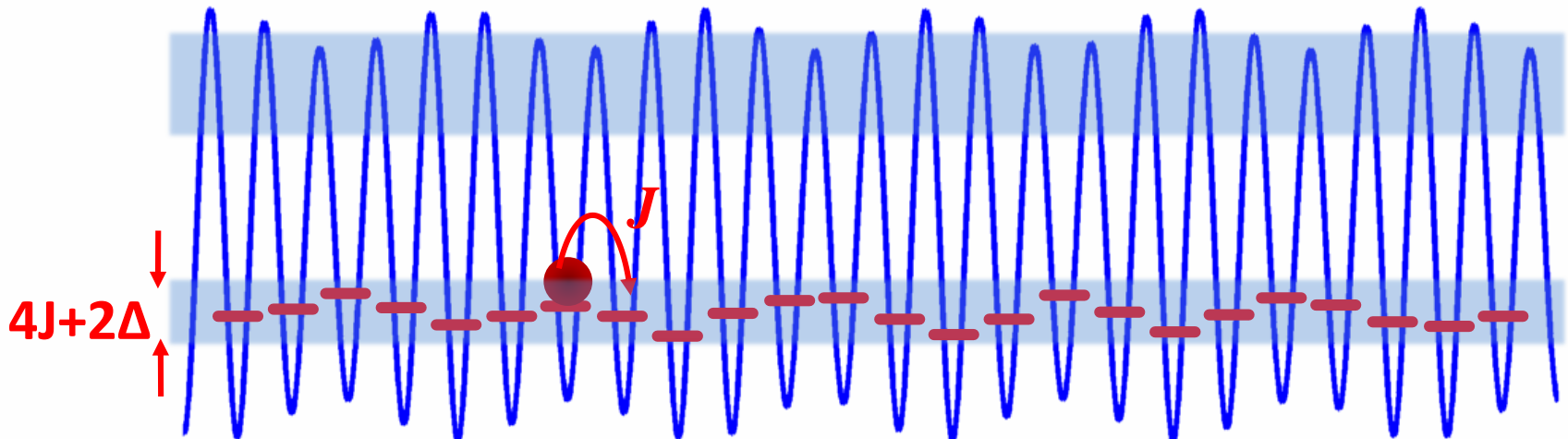
Quasiperiodic potential:  
Aubry-André Hamiltonian

$$H = -J \sum_i (b_i^\dagger b_{i+1} + h.c.) + \Delta \sum_i \cos(2\pi\beta i) n_i$$

$\beta = \frac{\lambda_1}{\lambda_2}$

Set  $J$  by fixing the strength of the primary lattice

Tune disorder by varying the strength of the secondary lattice

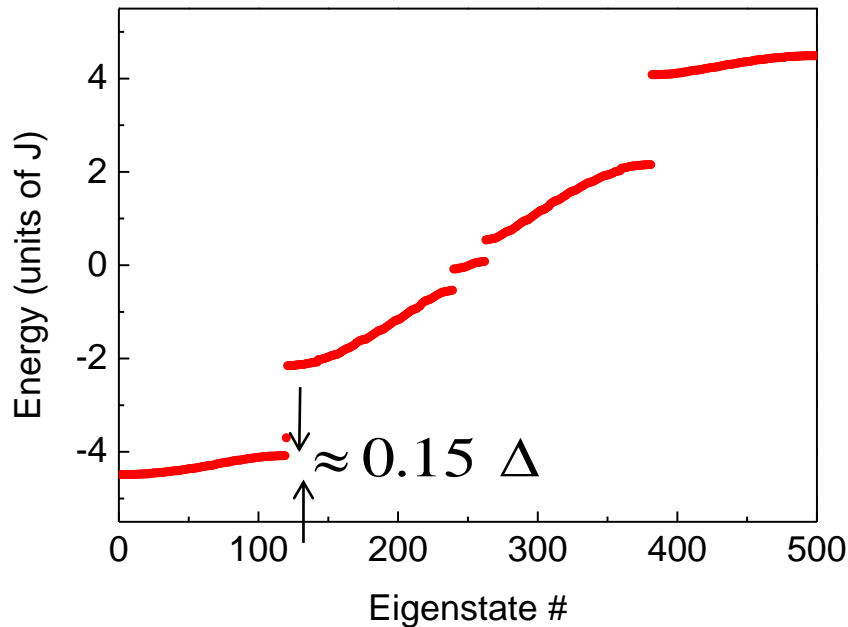


# Anderson localization in the quasiperiodic potential

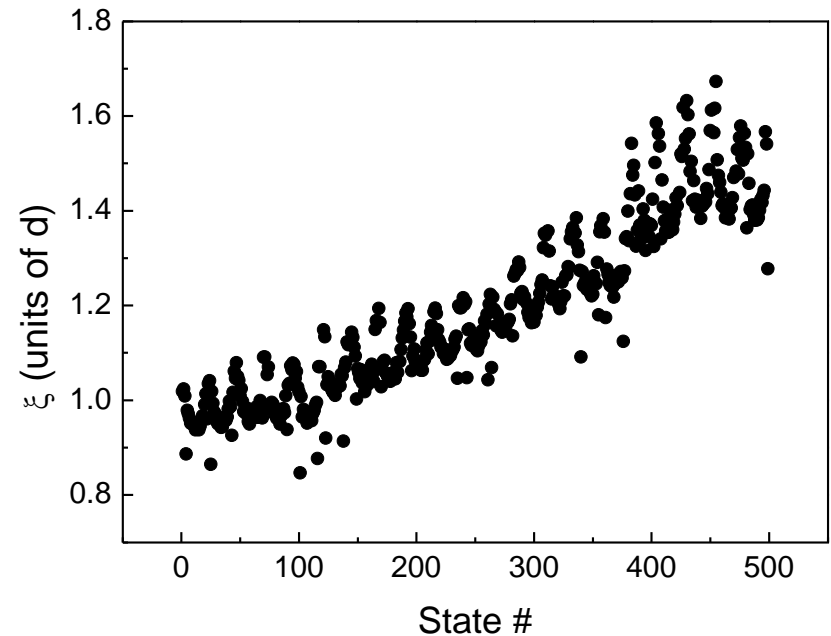
$$H = -J \sum_i (b_i^\dagger b_{i+1} + h.c.) + \Delta \sum_i \cos(2\pi\beta i) n_i$$

**Quasiperiodic potential:** localization transition at  $\Delta = 2J$

Eigenvalues: miniband structure



Eigenfunction: exponential decay



Short, uniform localization length:

$$\xi \approx d / \log(\Delta / 2J)$$

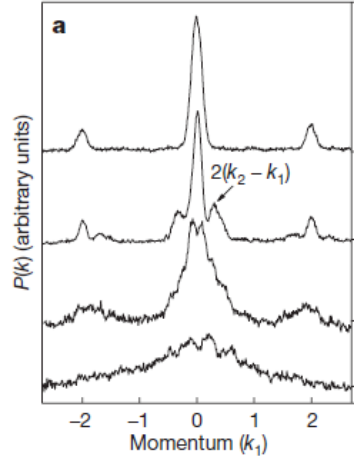
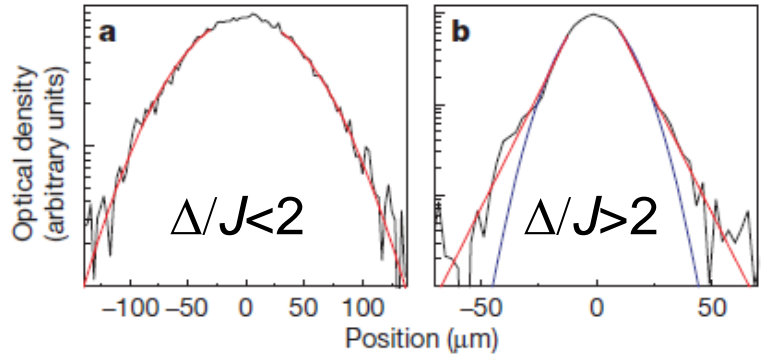


# Anderson localization in the quasiperiodic potential

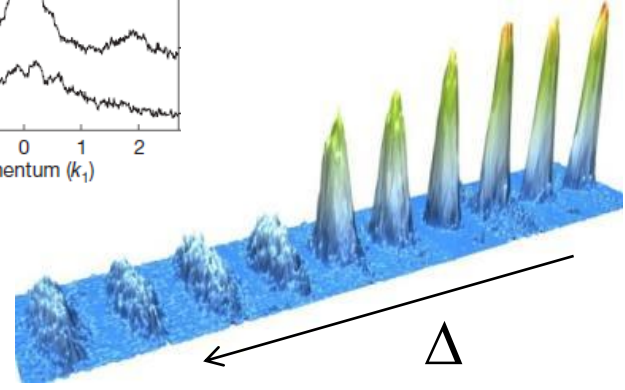
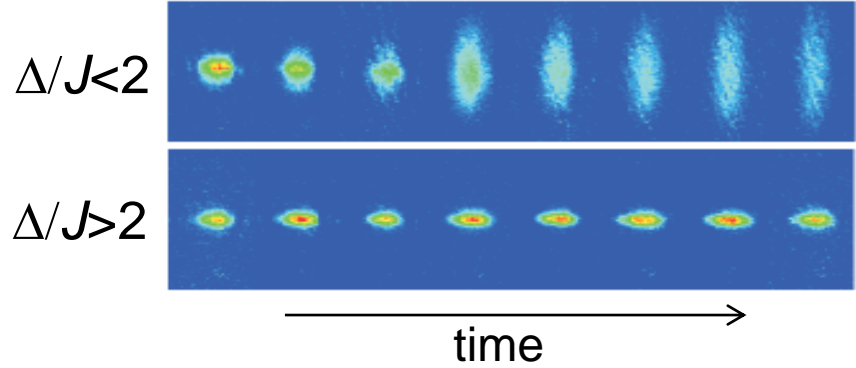
$$H = -J \sum_i (b_i^\dagger b_{i+1} + h.c.) + \Delta \sum_i \cos(2\pi\beta i) n_i$$

**Quasiperiodic potential:** localization transition at  $\Delta = 2J$

$|\Psi(x)|^2$  Density distribution:



$|\Psi(k)|^2$   
Momentum distribution



# Anderson localization in the quasiperiodic potential

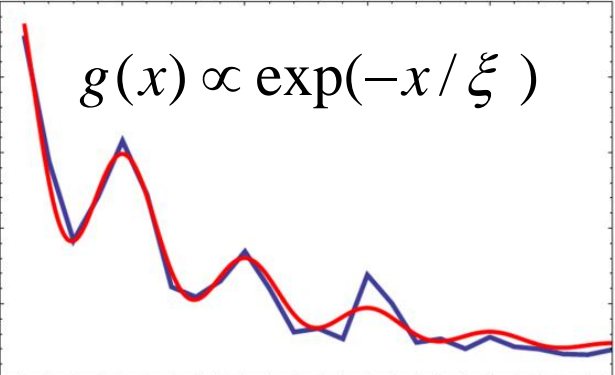
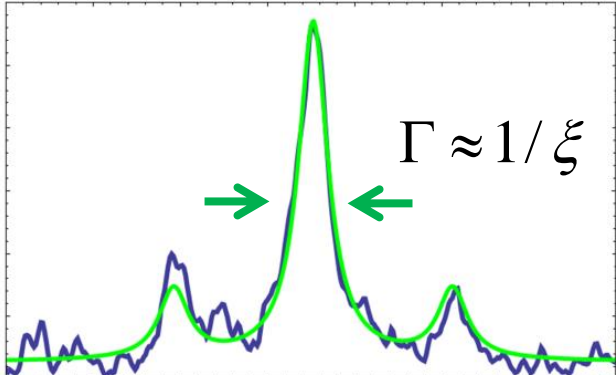
$|\Psi(k)|^2$ 

 $\xrightarrow{\text{FT}}$ 
 $g(x) = \int dx' \langle \Psi^+(x) \Psi(x+x') \rangle$

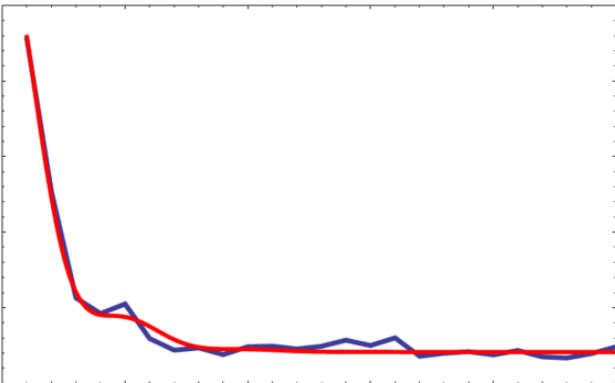
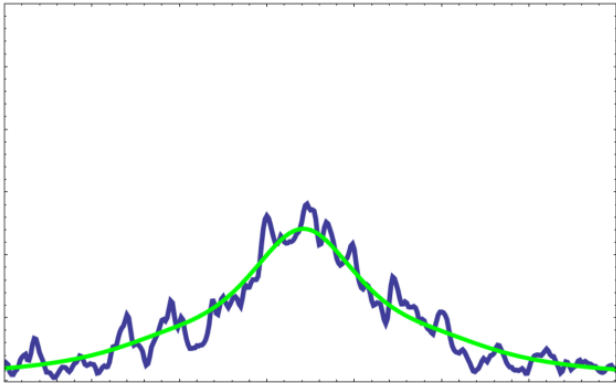
Momentum distribution

Spatially averaged correlation function

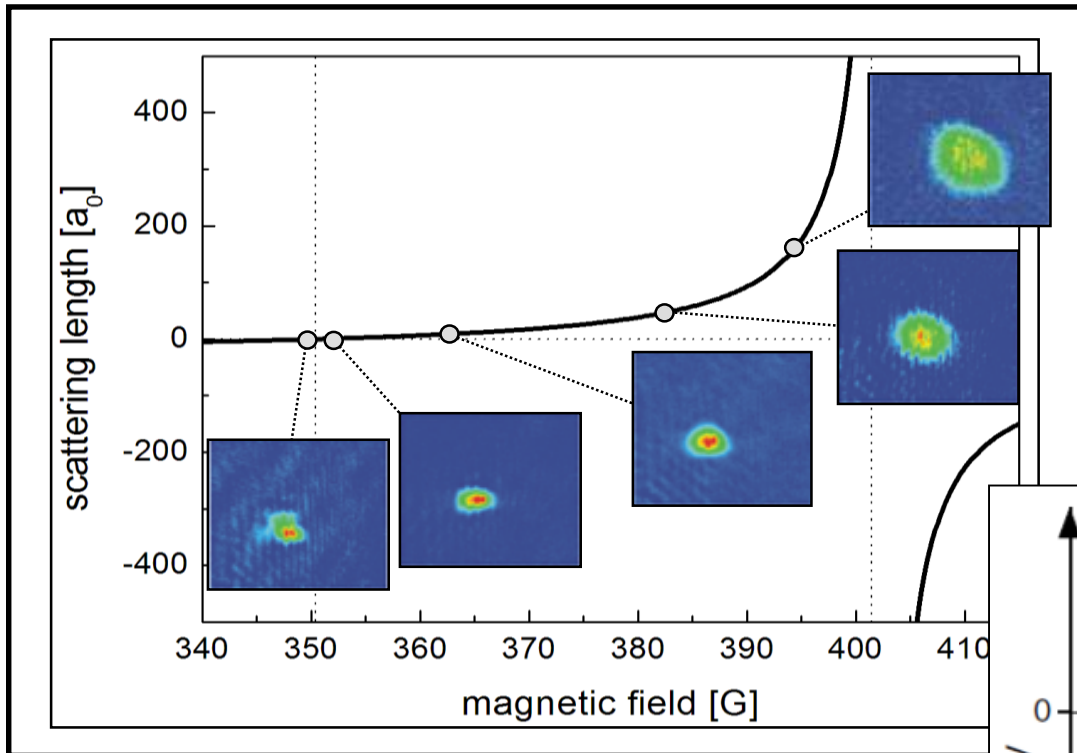
$\Delta < 2J$   
Coherent extended state



$\Delta > 2J$   
Incoherent localized state



# Interaction tuning: Feshbach resonance

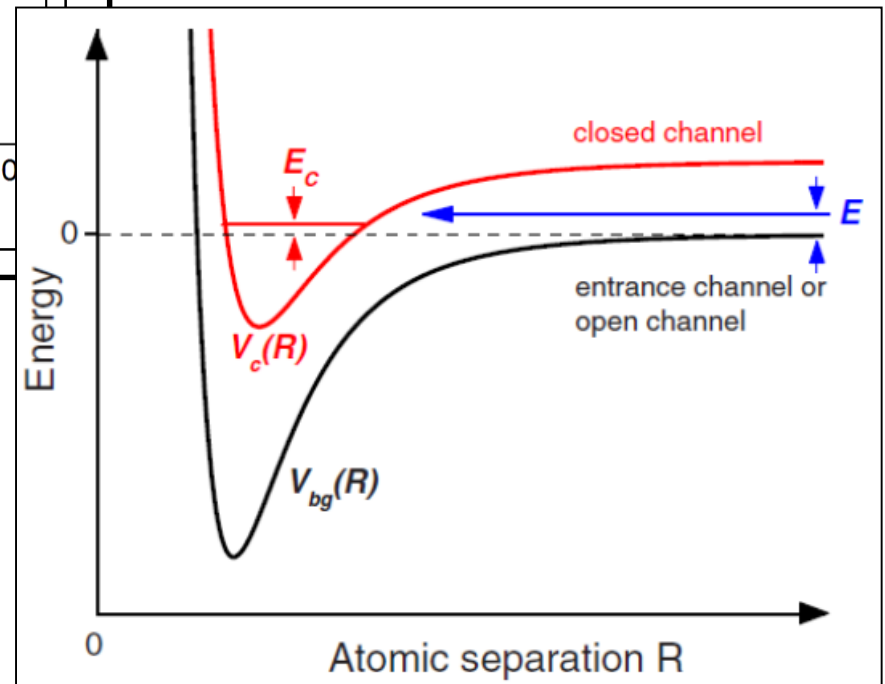


$^{39}\text{K}$  atoms:  
broad magnetic Feshbach  
resonance

Two-body interaction potential:  
contact interaction

$$v(r-r') = \frac{4\pi\hbar^2}{m} a \delta(r-r')$$

s-wave scattering length



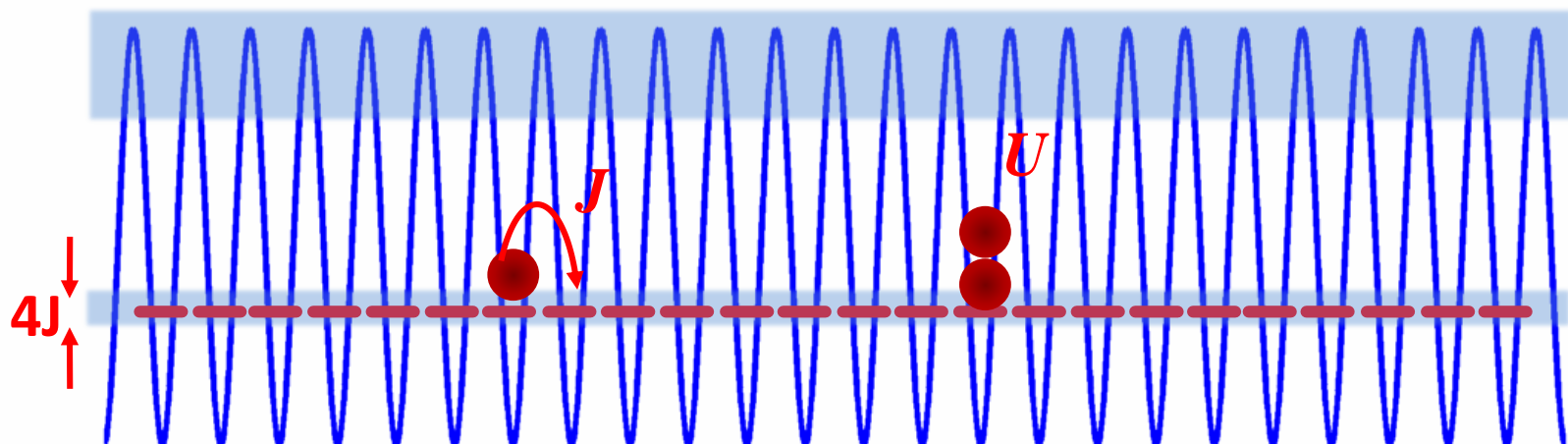
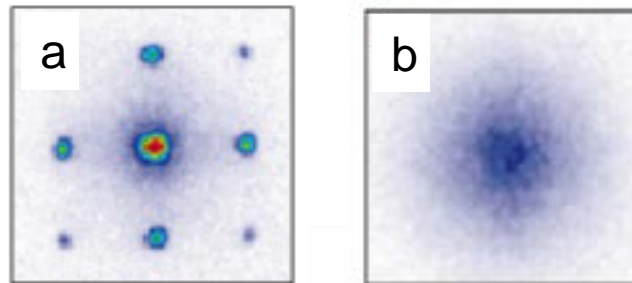
# Interaction in the lattice

Bose-Hubbard Hamiltonian

$$H = -J \sum_i (b_i^\dagger b_{i+1} + h.c.) + \frac{U}{2} \sum_i n_i(n_i - 1)$$

$$U = \frac{4\pi\hbar^2}{m} a \int |\varphi(x)|^4 d^3x$$

Mott insulator transition



# Disordered Bose-Hubbard

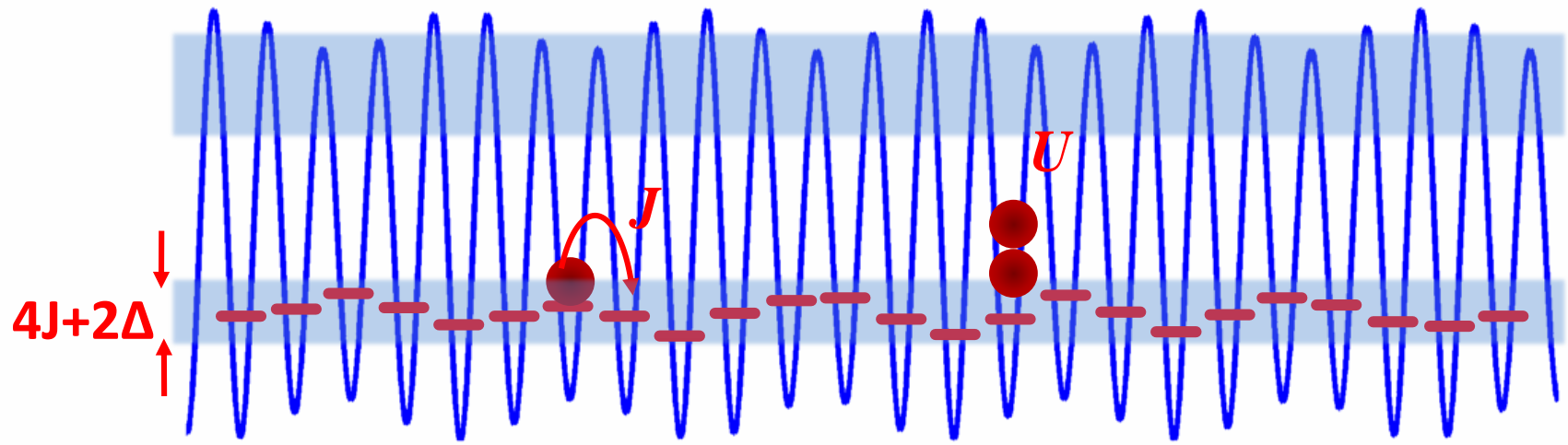
**$^{39}\text{K}$  bosonic atoms with controlled interparticle repulsive interactions in a quasiperiodic lattice potential**

$$H = -J \sum_i (b_i^\dagger b_{i+1} + h.c.) + \Delta \sum_i \cos(2\pi\beta i) n_i + \frac{U}{2} \sum_i n_i(n_i - 1)$$

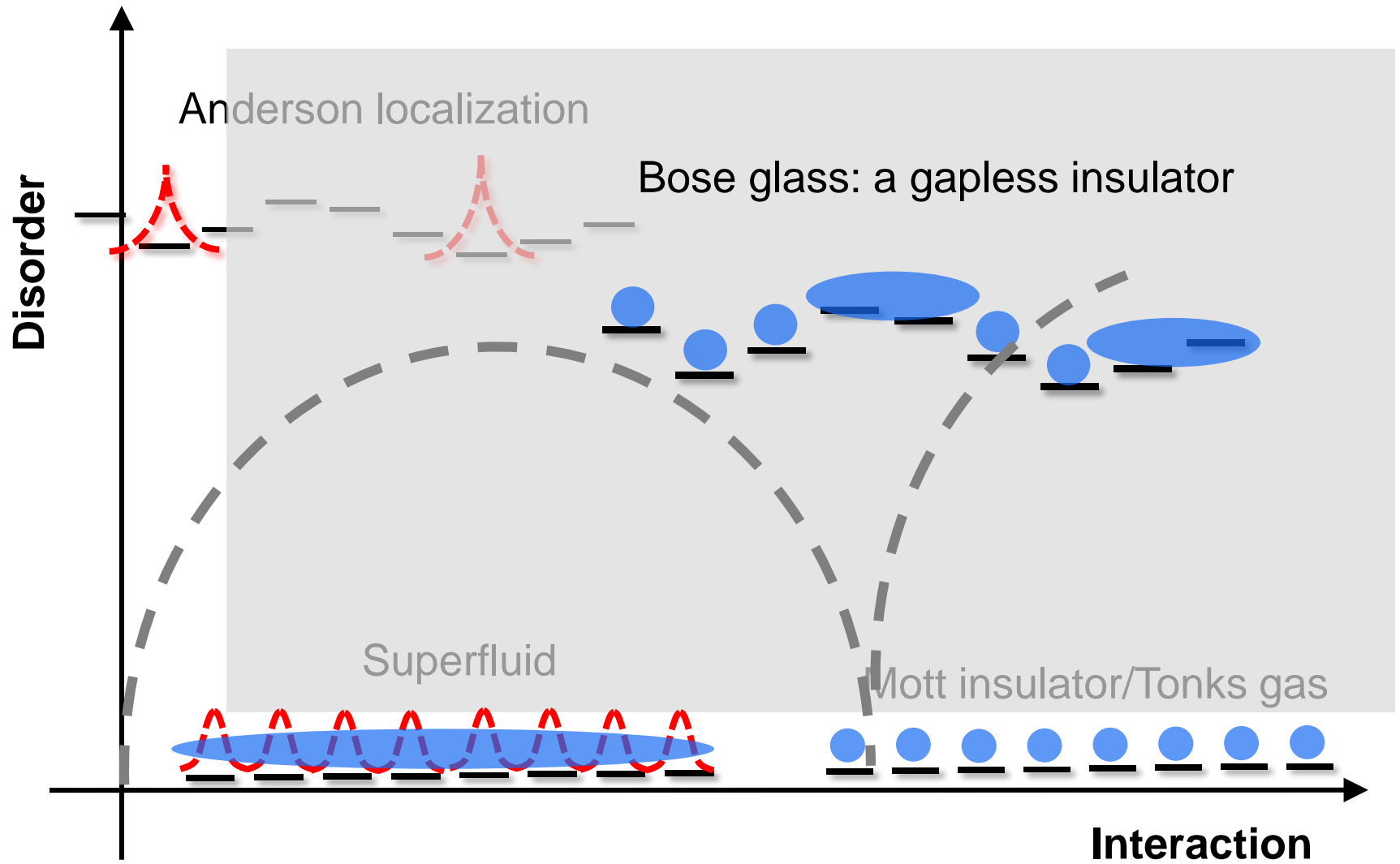
Set  $J$  by fixing the strenght of the primary lattice

Tune disorder by varying the strenght of the secondary lattice

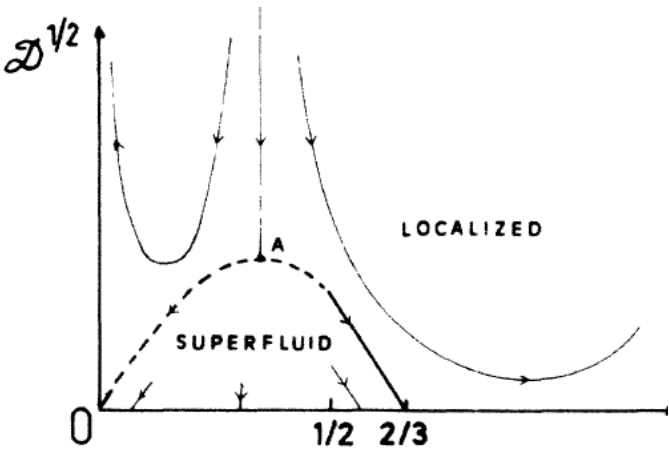
Tune interaction by varying the external magnetic field



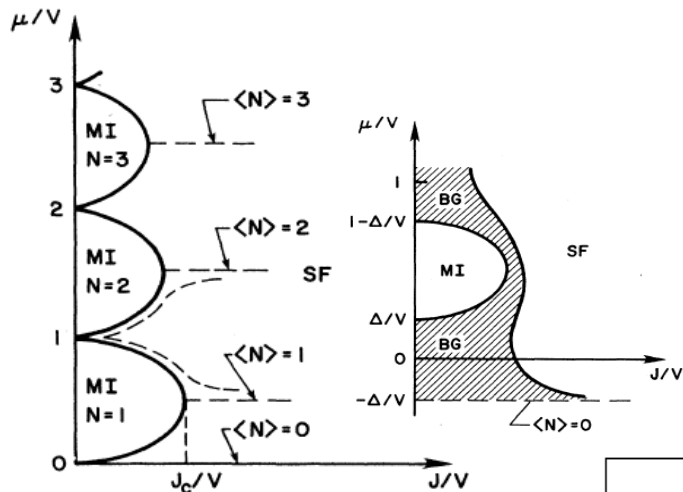
# Quantum phases across the $\Delta$ -U diagram



# 1D disordered interacting bosons

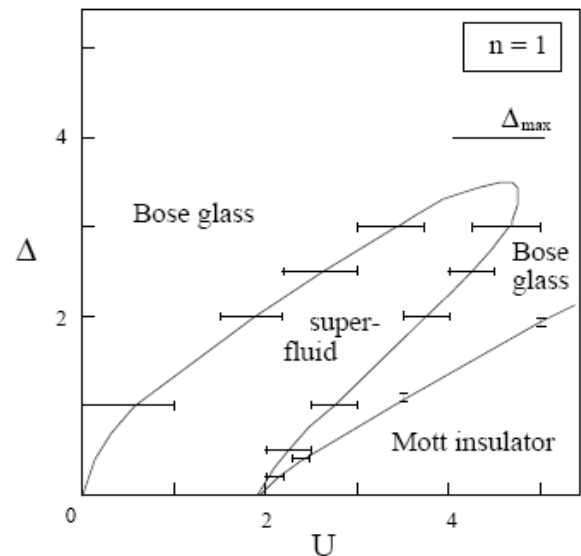


Giamarchi & Schultz,  
PRB 37 325 (1988)



Fisher et al  
PRB 40, 546 (1989)

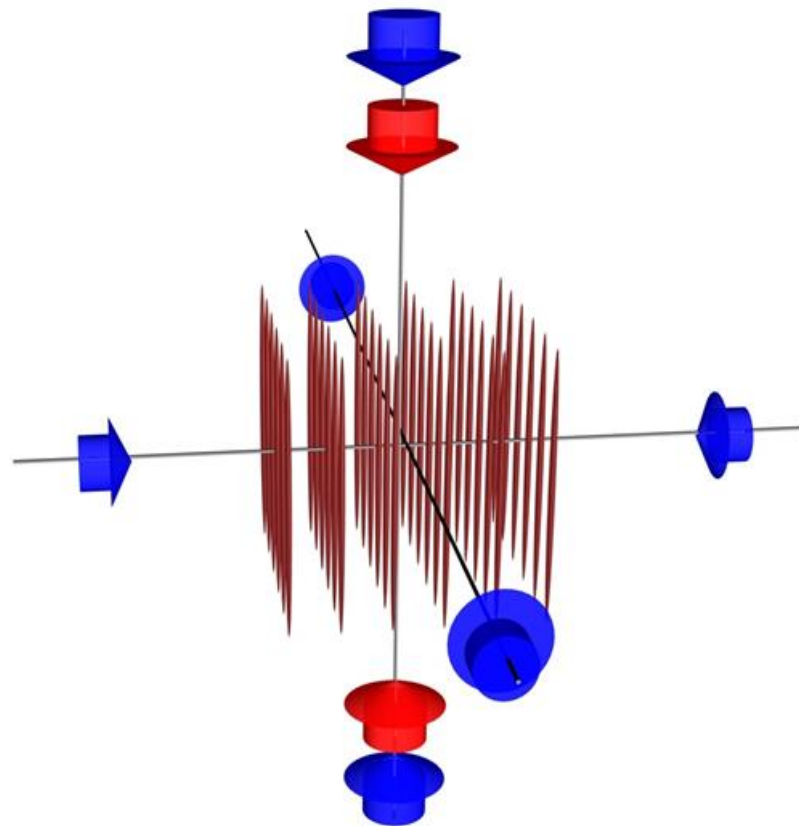
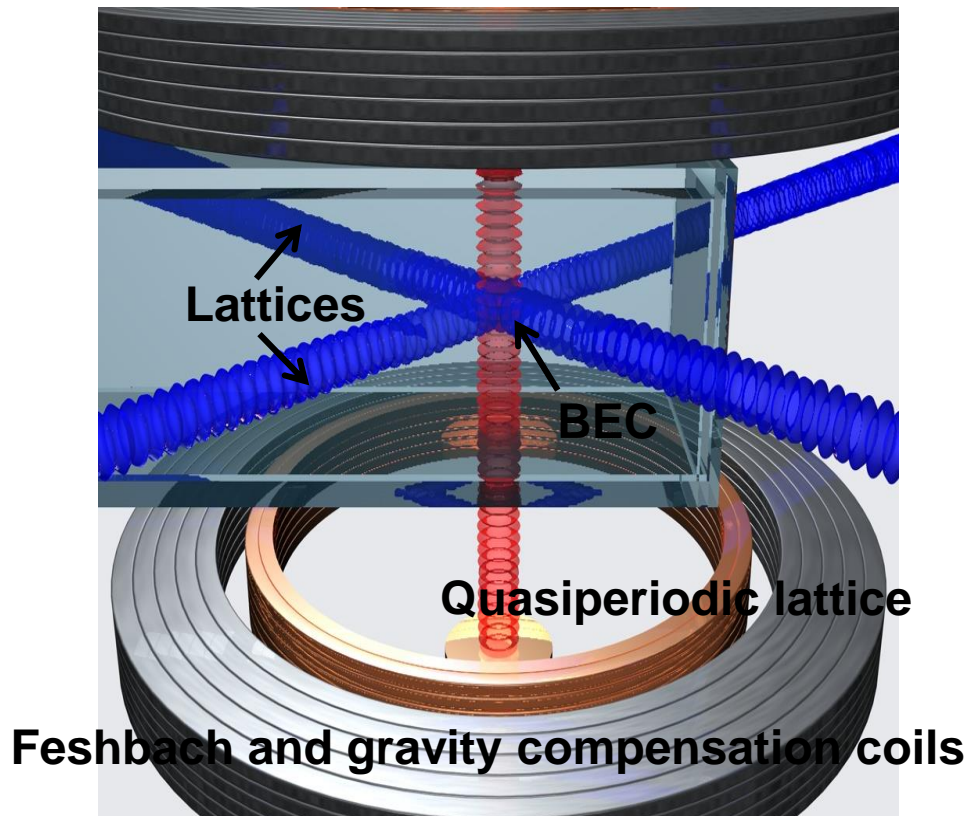
One dimensional bosons are the prototype disordered systems, with an established theoretical framework.



Rapsch, et al. EPL 46 559 (1999)

And many others (1988-2013)...

# Experimental configuration: 1D systems



Radial energy much larger than any other energy:  $\nu_r=50$  kHz;  $J/h=100$  Hz



# Experimental results

- Investigation of the  $\Delta$ -U diagram and comparison with zero-temperature theory

C. D'Errico, E. Lucioni et al., submitted

- Transport instability at the fluid-insulator transition

L. Tanzi et al., Phys. Rev. Lett. (2013)

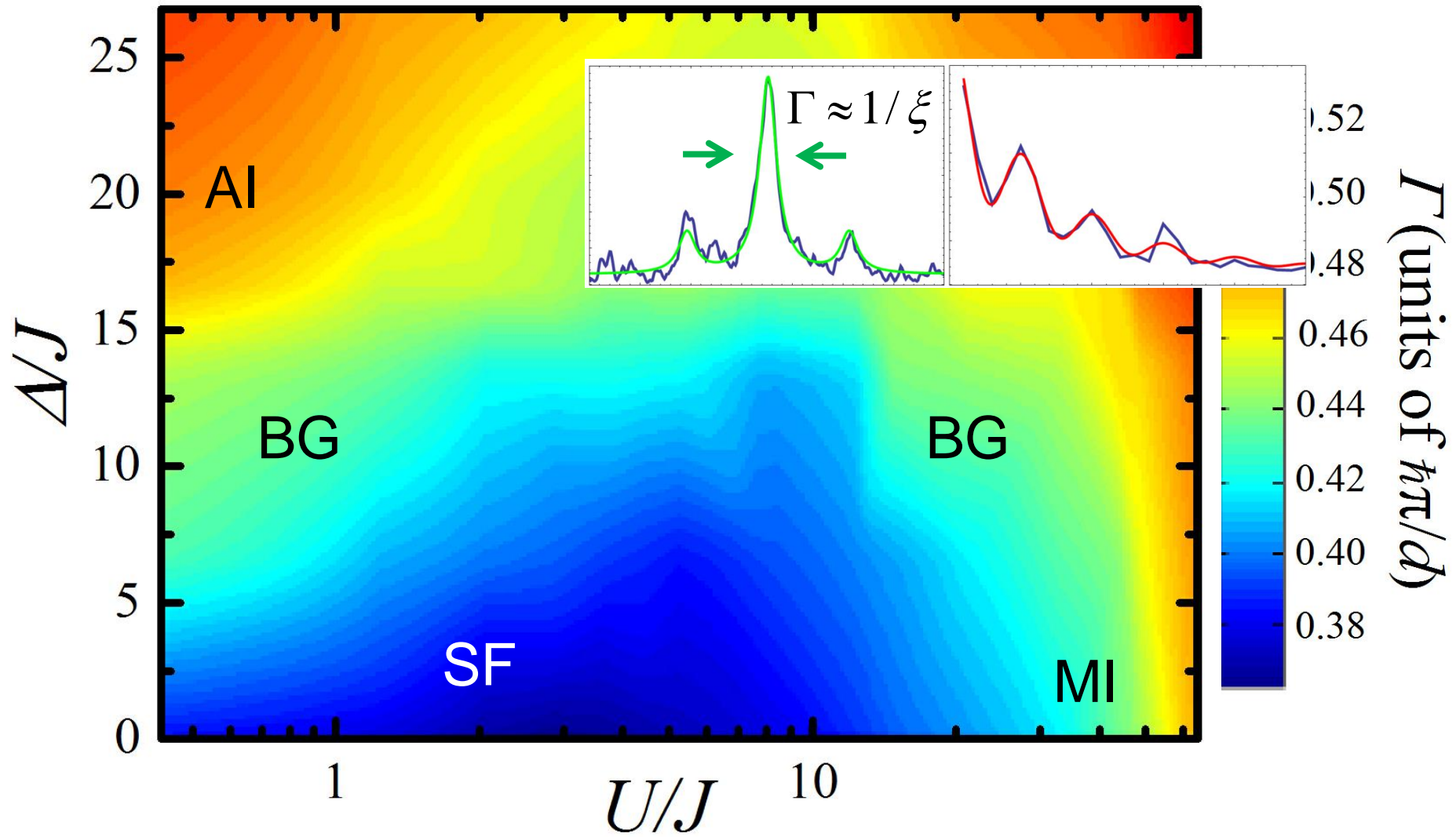
- Induced expansion dynamics in a disorder-localized system

E. Lucioni et al., Phys. Rev. Lett. 106, 23 (2011)

E. Lucioni et al., Phys. Rev. E 87, 042922 (2013)

C. D'Errico et al., New J. Phys.15, 045007 (2013)

# $\Delta$ -U diagram: coherence measurements



- Finite size systems  $\longrightarrow$  Transitions  $\rightarrow$  Crossovers  
 - Non-uniform density

# Incoherent phases are insulating: transport

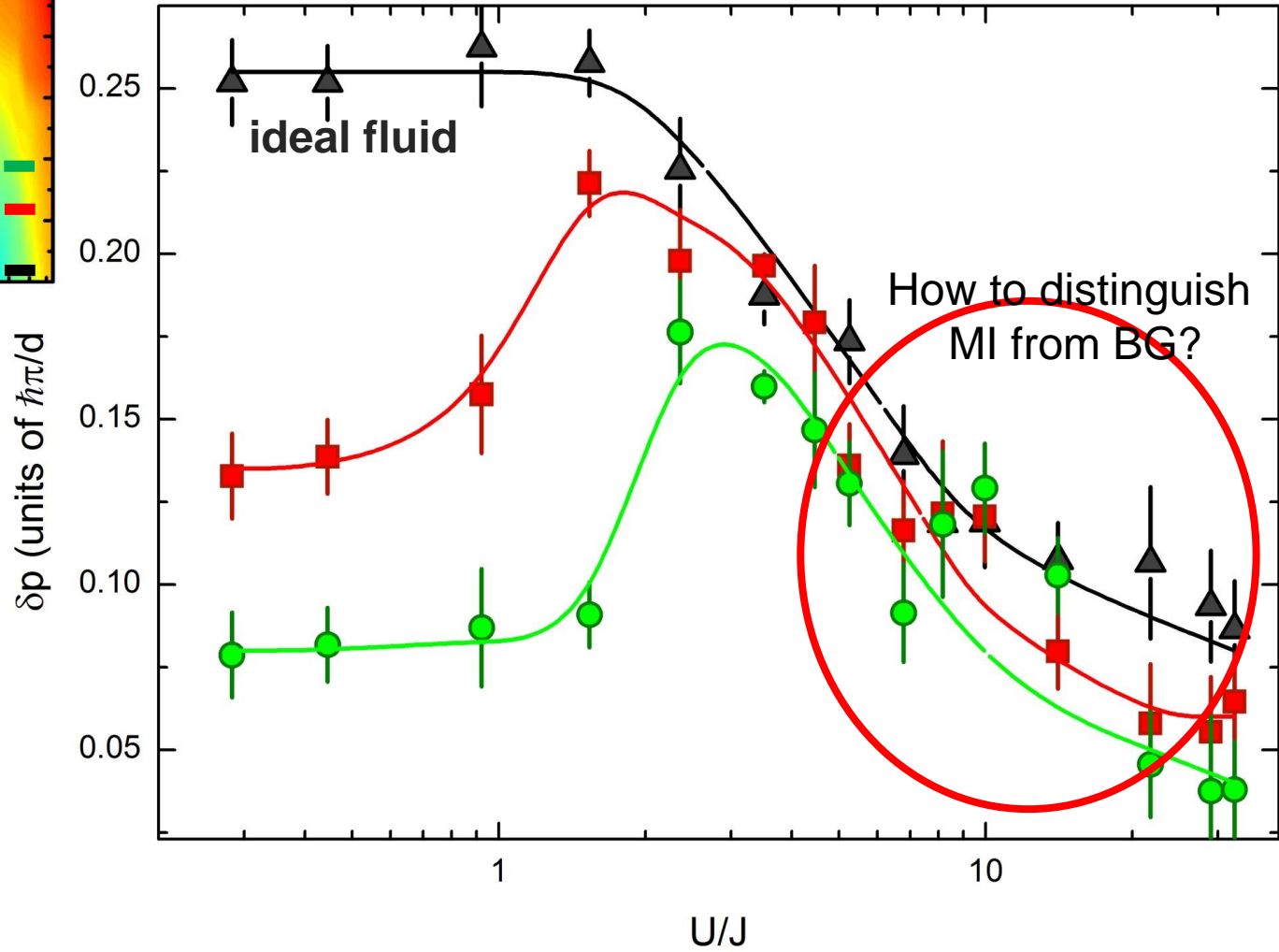
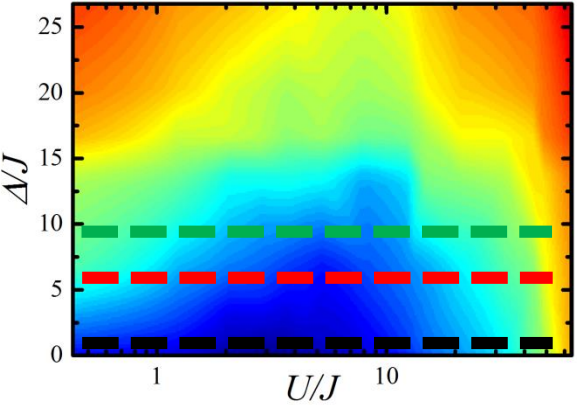
prepare in equilibrium



kick, wait 0.8ms



free expansion

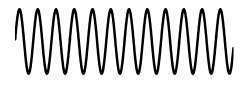


# Evidence of a strongly correlated BG: spectra

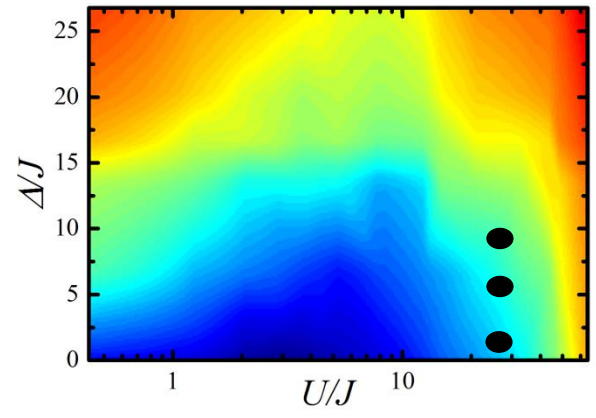
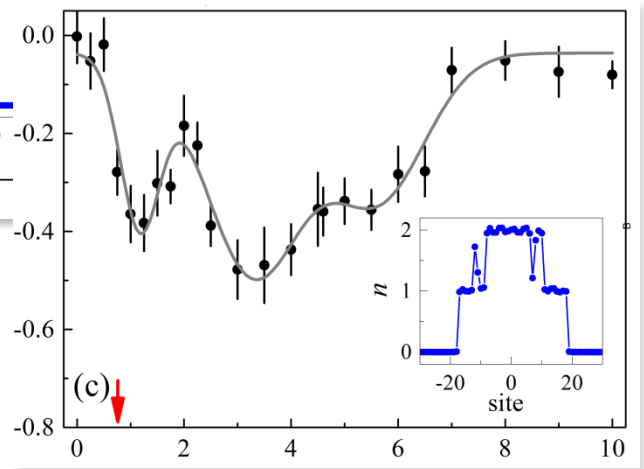
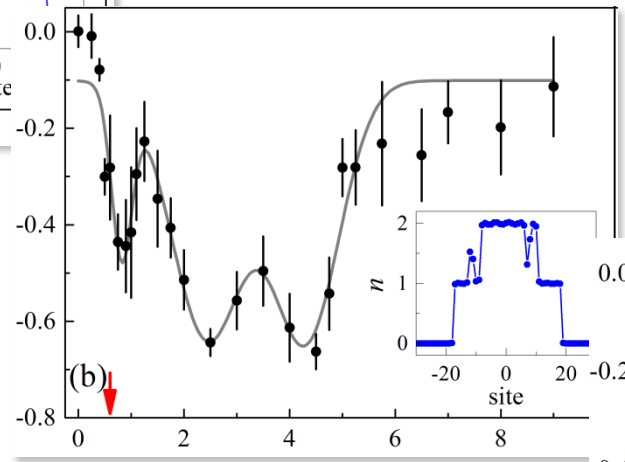
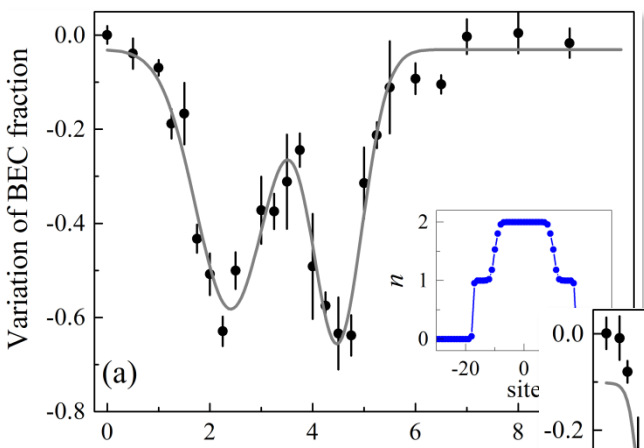
prepare in equilibrium



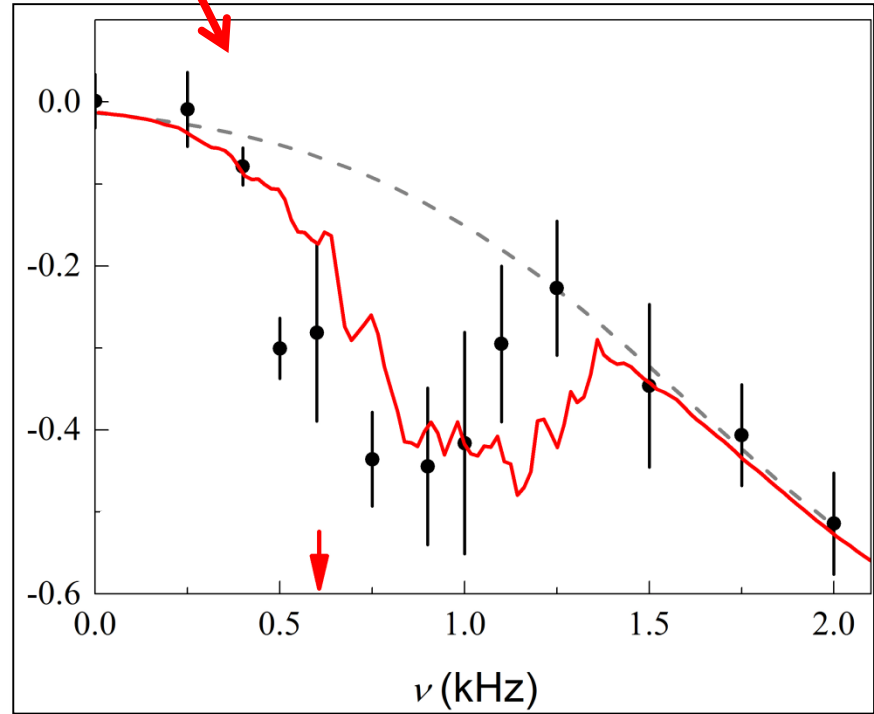
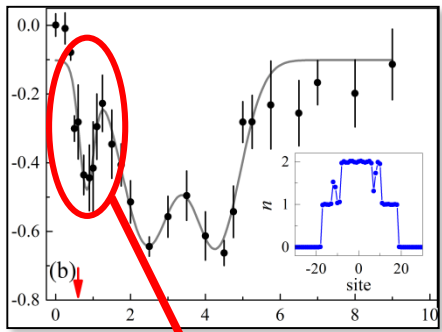
main lattice modulation  
(15%, 200ms)



“energy” measurement

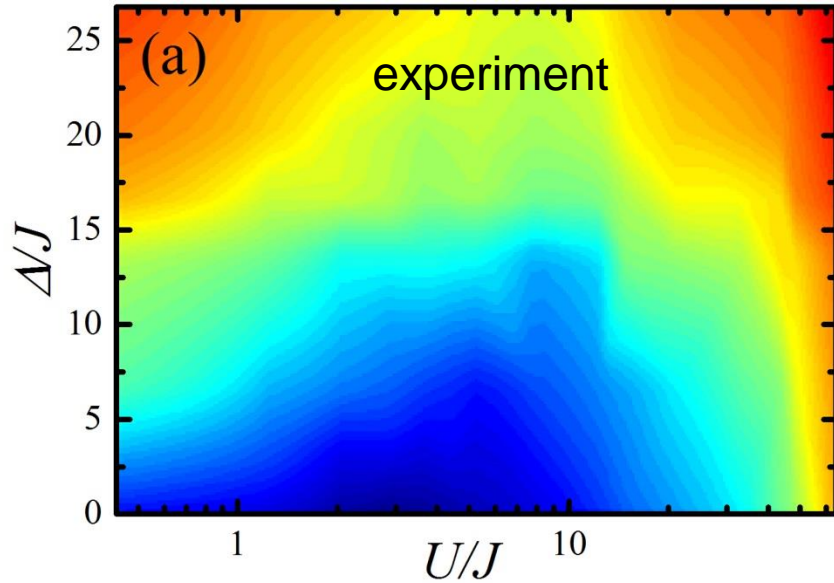


# Evidence of a strongly correlate BG: spectra



We calculate the energy absorption rate:  
Response of **non-interacting fermions**:  
the strongly-correlated SF is Anderson-localized by disorder.

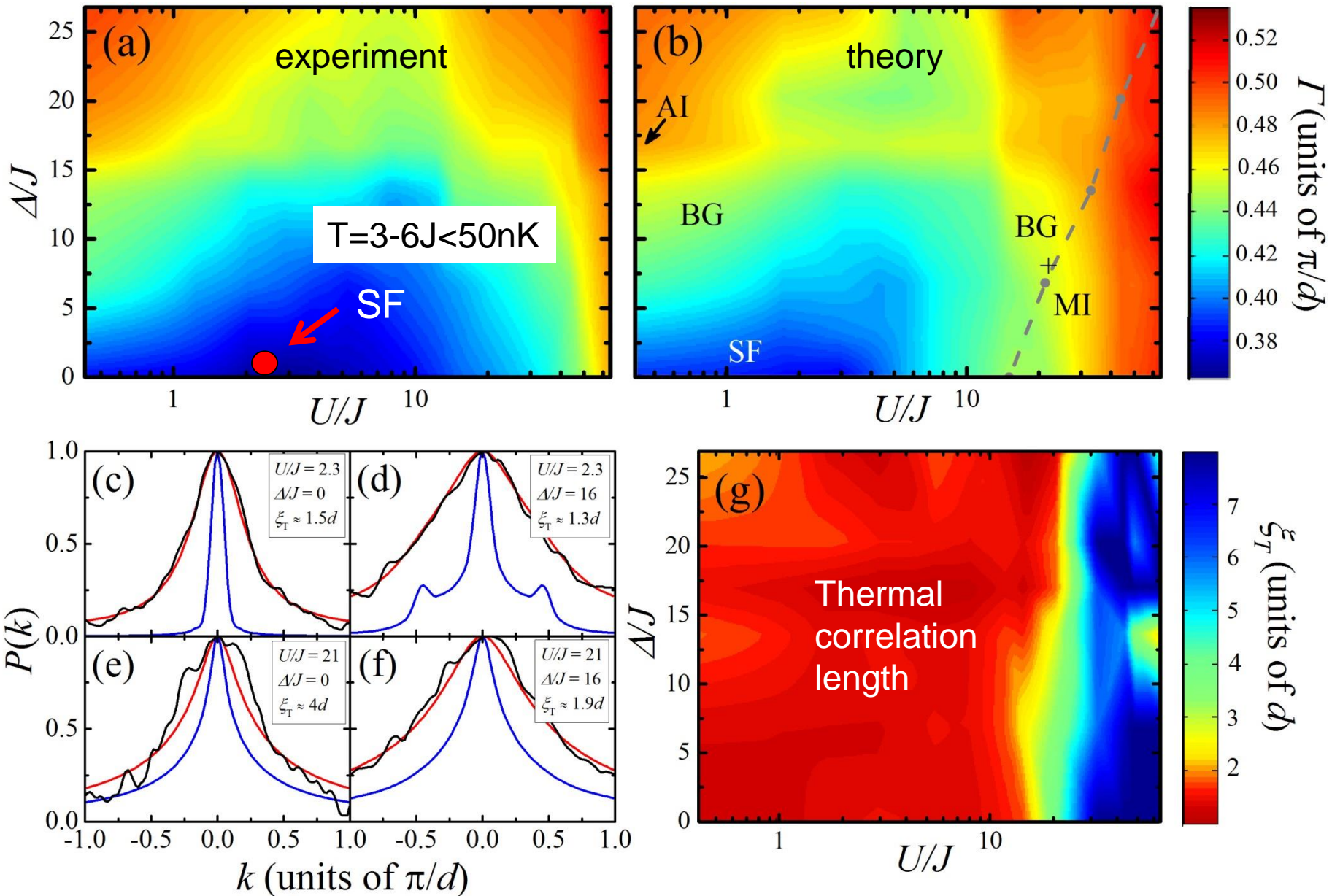
# Connection between zero-T theory and experiment



We fit a thermal broadening of  $P(k)$  that corresponds to an exponential decay of the correlation function  $g(x)$  with thermal length  $\xi_T$

$$g_T(x) \propto \exp(-x / \xi_T)$$

# Connection between zero-T theory and experiment



# Experimental results

- Investigation of the  $\Delta$ -U diagram and comparison with zero-temperature theory

C. D'Errico, E. Lucioni et al., submitted

*Reentrance of the Bose glass from coherence measurements*  
*Evidence of a strongly correlated BG from the excitation spectra*  
*Connection with zero-temperature phase diagram*

- Transport instability at the fluid-insulator transition:

L. Tanzi et al., Phys. Rev. Lett. (2013)

- Induced expansion dynamics in a disorder-localized system:

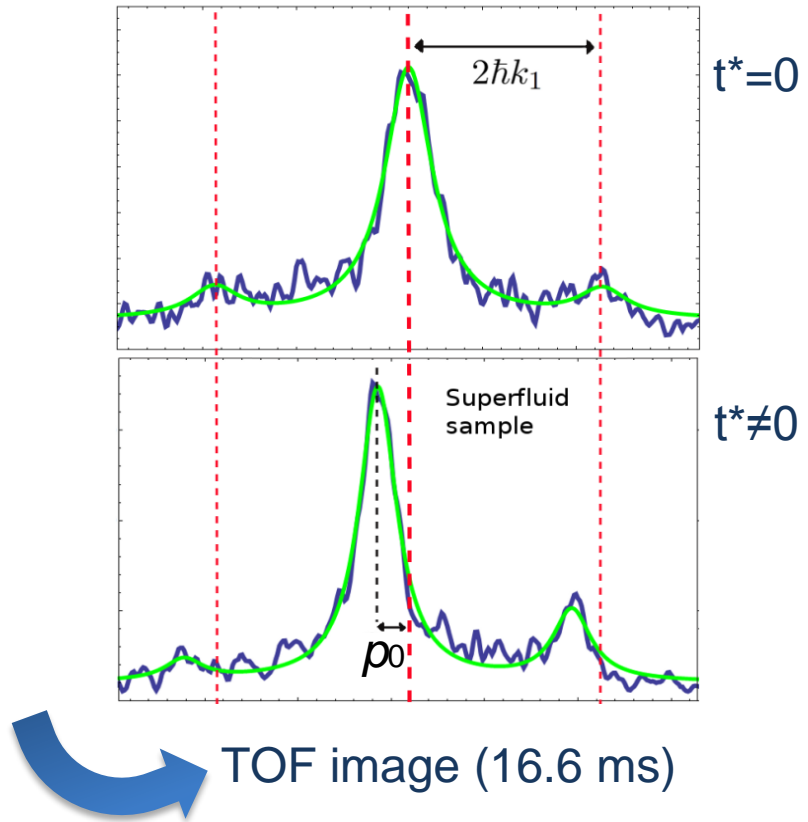
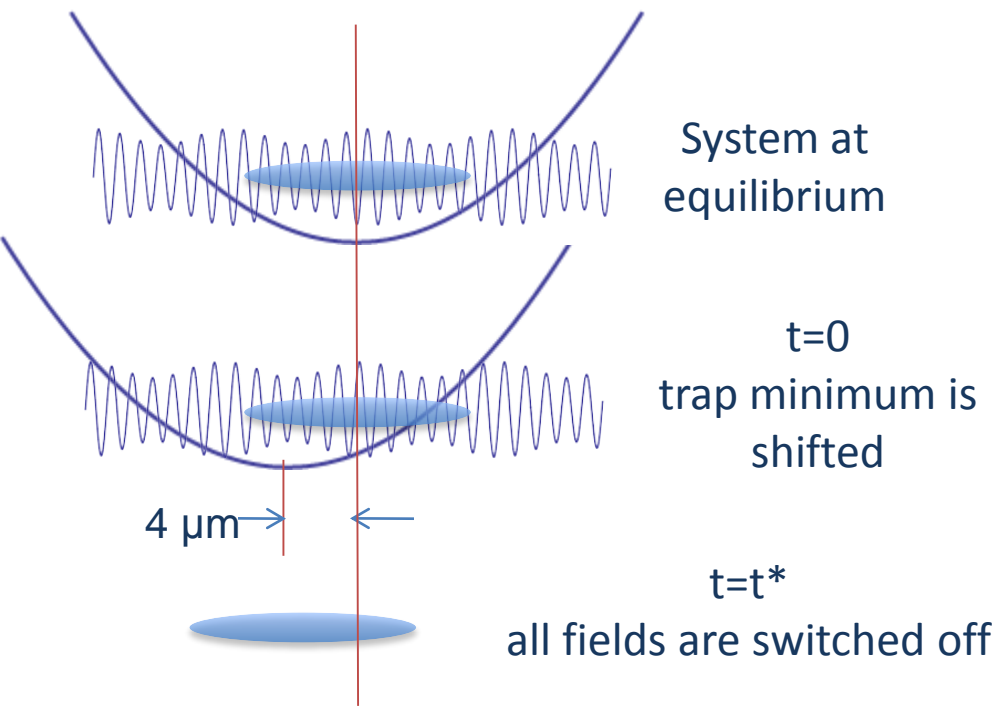
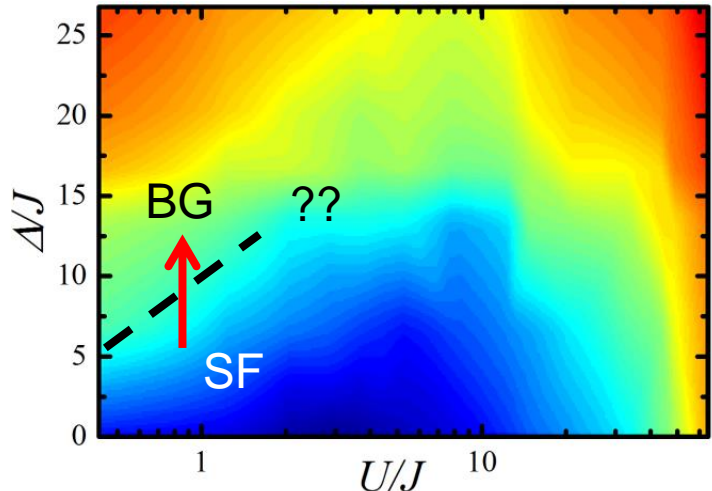
E. Lucioni et al., Phys. Rev. Lett. 106, 23 (2011)

E. Lucioni et al., Phys. Rev. E 87, 042922 (2013)

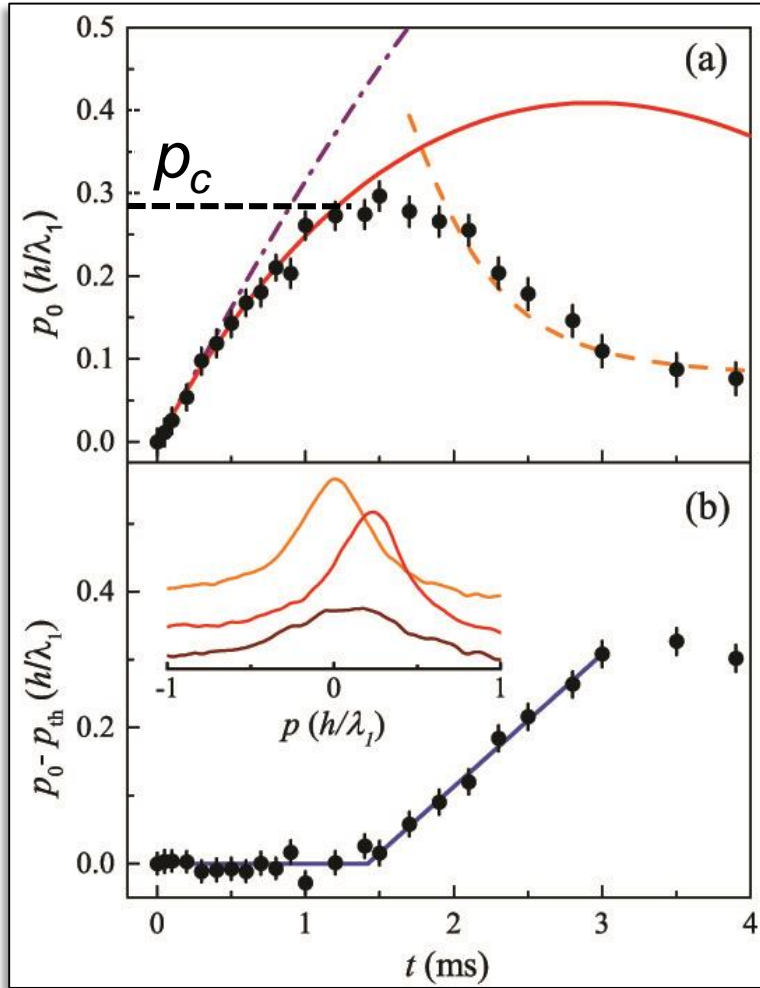
C. D'Errico et al., New J. Phys. 15, 045007 (2013)



# Transport measurements at the SF-BG transition



# Critical momentum in the clean lattice



$U/J=1.5$   
 $n=3.5$

Existence of a critical moment:

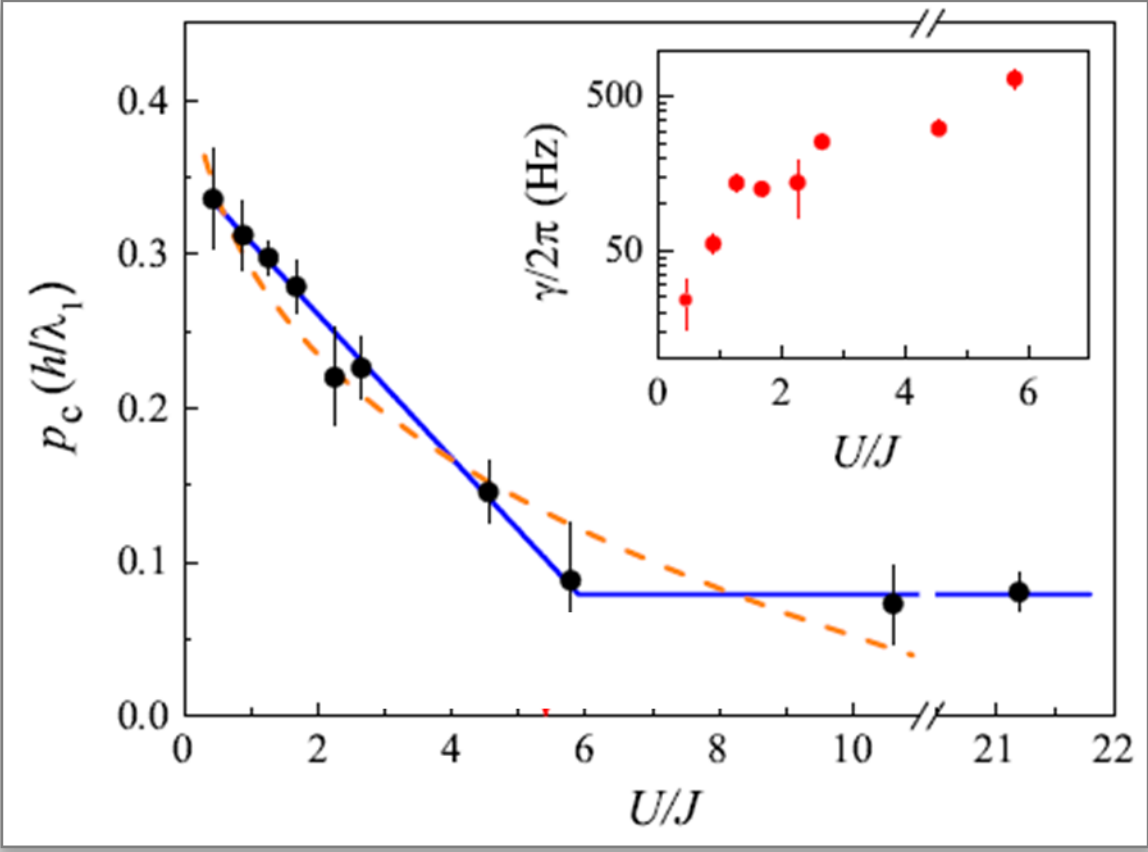
For vanishing interaction  $p_c$  is expected to be 0.5  
→ dynamical instability

Increasing the interactions we approach a phase transition (SF-MI) and there is a p-dependent damping due to Quantum phase slips

$$\Gamma_Q \propto \exp\left(-1.7 \sqrt{\frac{nJ}{U}} \left(\frac{\pi}{2} - \frac{p\lambda_1}{2\hbar}\right)^{\frac{5}{2}}\right)$$

E. Altman et al, Phys. Rev. Lett. 95, 020402 (2005).  
 J. Schachenmayer, G. Pupillo, and A. J. Daley, New J. Phys. 12, 025014 (2010).  
 I. Danshita and A. Polkovnikov, Phys. Rev. A 85, 023638 (2012).  
 I. Danshita, Phys. Rev. Lett. 111, 025303 (2013).

# Critical momentum in the clean lattice



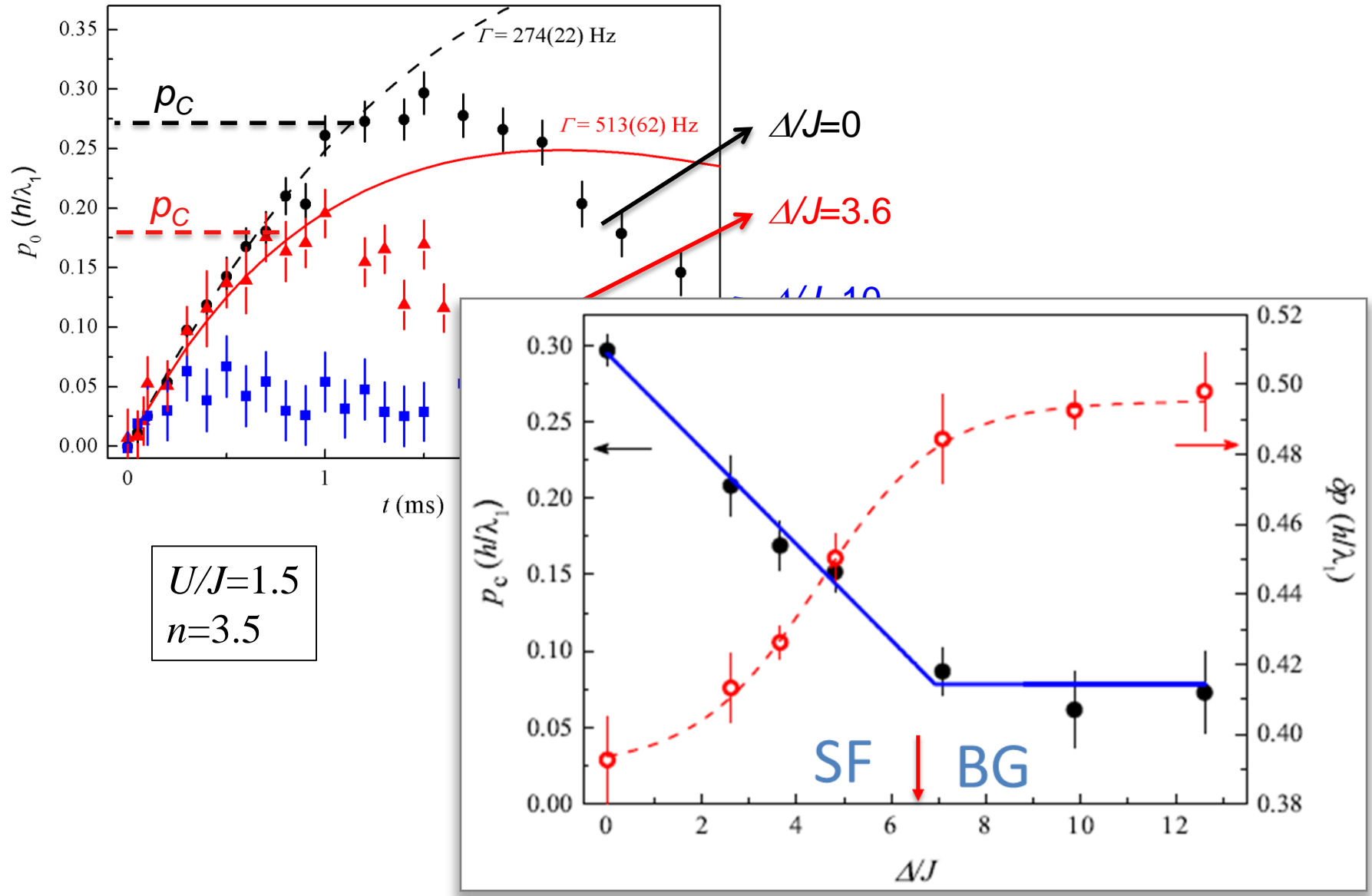
Quantum phase slips

$$\Gamma_Q \propto \exp\left(-1.7 \sqrt{\frac{nJ}{U}} \left(\frac{\pi}{2} - \frac{p\lambda_1}{2\hbar}\right)^{2.5}\right)$$

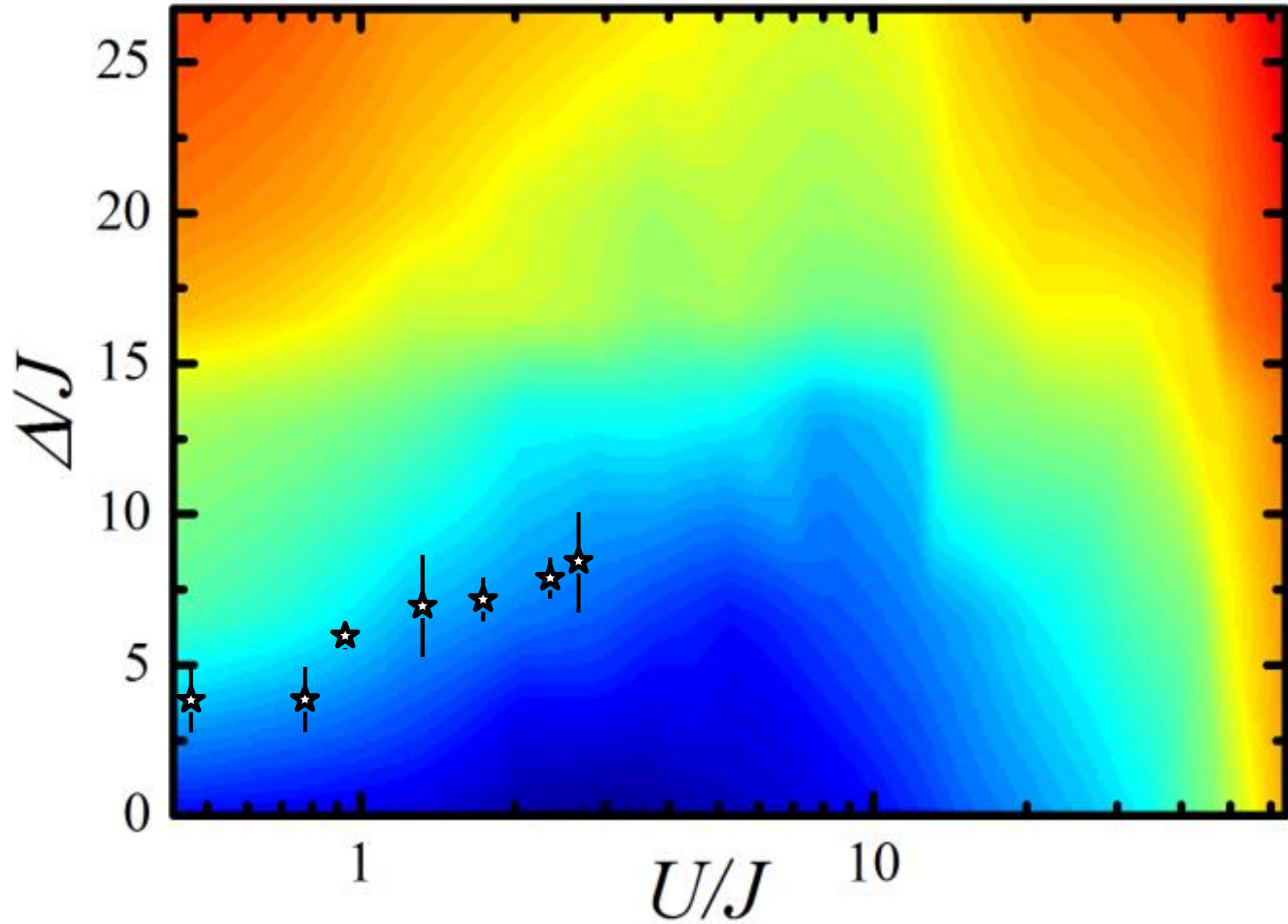
We identify the critical  $U/J$  for the MI «transition»  
 the one for which we reach the minimum  
 detectable  $p_c$   
**insulator** → **vanishing  $p_c$**

Similar experiment in 3D: J. Mun et al., Phys. Rev. Lett. 99, 150604 (2007).

# Critical momentum in the disordered lattice



# “Experimental SF-BG transition”



# Experimental results

- Investigation of the  $\Delta$ -U diagram and comparison with zero-temperature theory

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*Reentrance of the Bose glass from coherence measurements*  
*Evidence of a strongly correlated BG from the excitation spectra*  
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L. Tanzi et al., Phys. Rev. Lett. (2013)

*Dominant role of quantum phase slips*  
*«experimental» transition: vanishing of critical momentum*

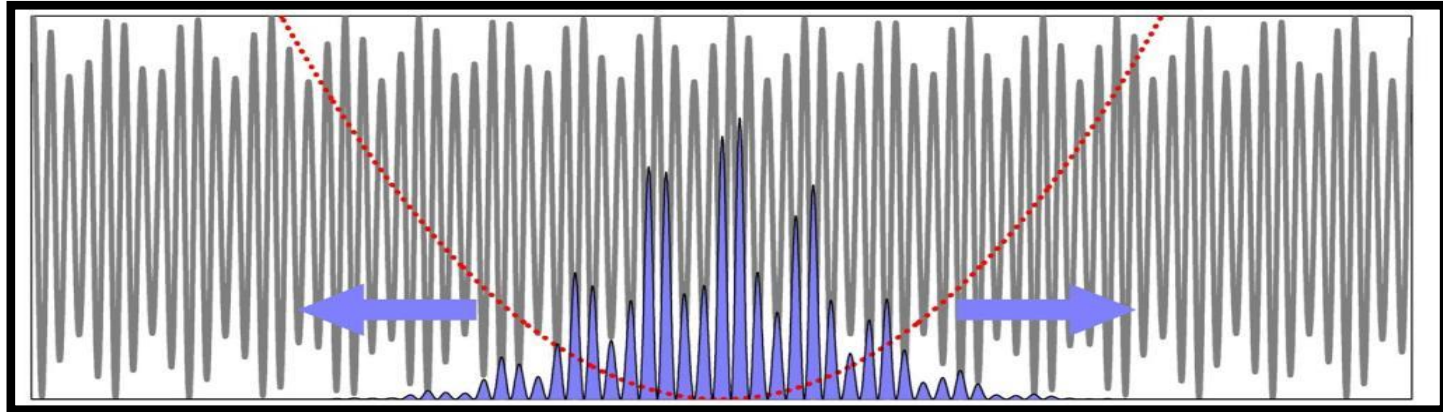
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E. Lucioni et al., Phys. Rev. Lett. 106, 23 (2011)

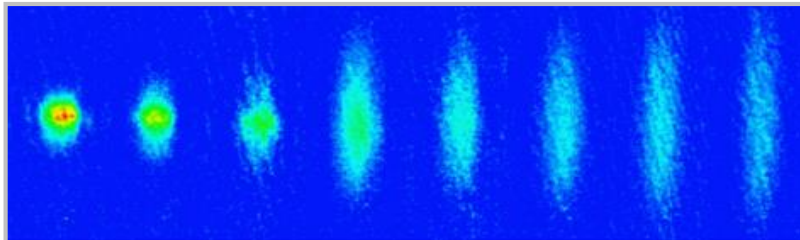
E. Lucioni et al., Phys. Rev. E 87, 042922 (2013)

C. D'Errico et al., New J. Phys. 15, 045007 (2013)

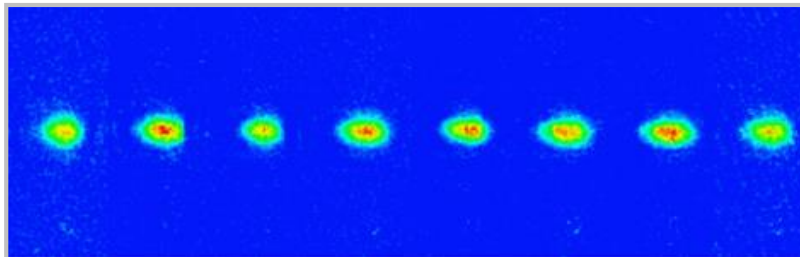
# Interaction-assisted expansion



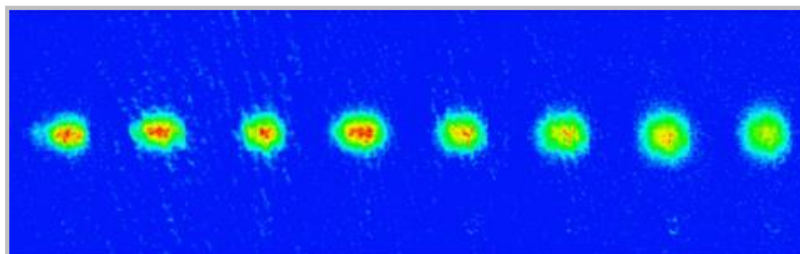
time



Clean lattice: ballistic expansion



Non-interacting sample  
in a disordered lattice: no expansion

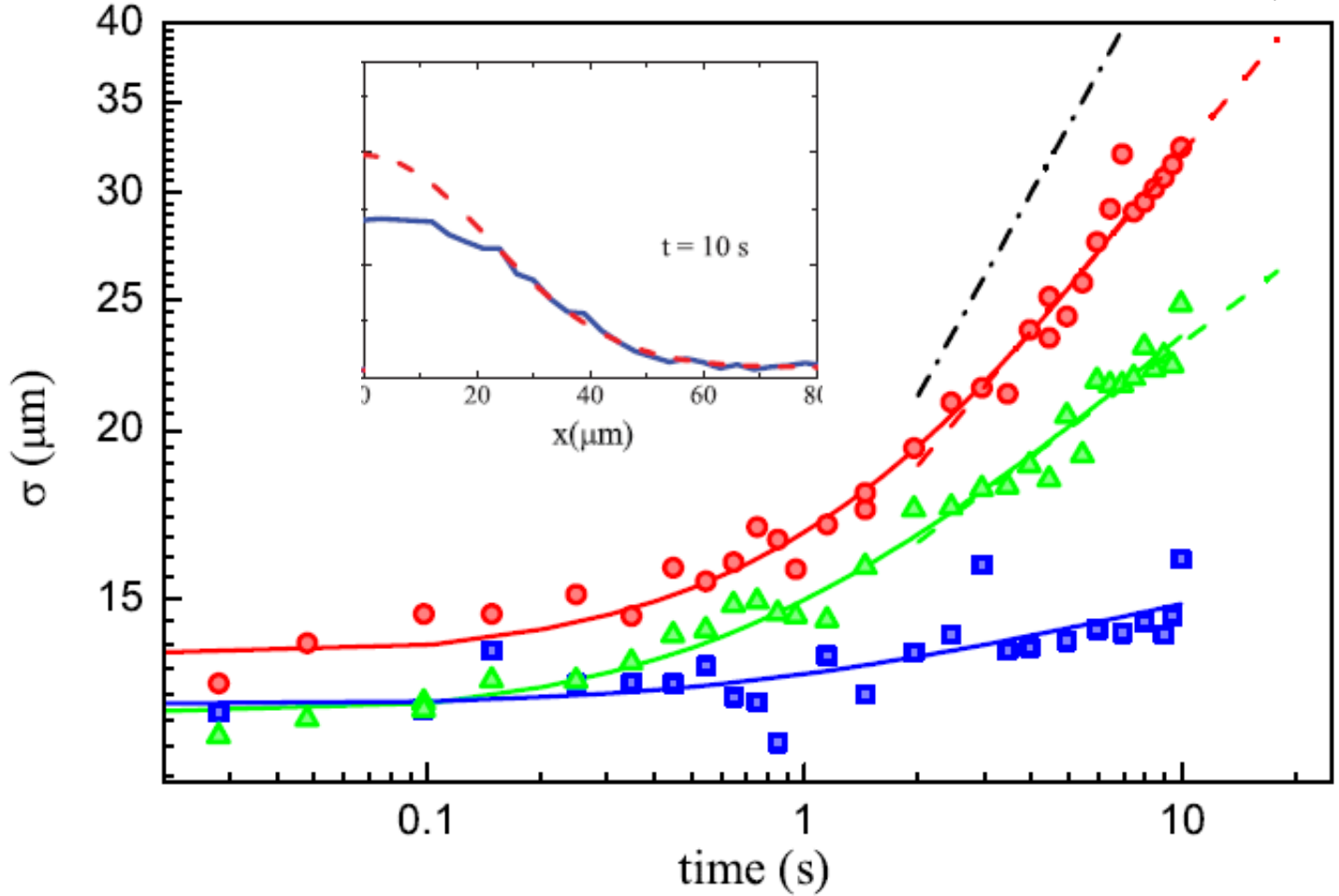


Interacting sample  
in a disordered lattice: slow expansion

# Expansion dynamics: anomalous diffusion

$$\sigma(t) \approx t^\alpha \quad \alpha < 0.5$$

D. L. Shepelyansky, Phys. Rev. Lett. 70, 1787 (1993)  
 S. Flach *et al*, Phys. Rev. Lett. 102, 024101 (2009)  
 M. Larcher *et al*. PRA **80**, 053606 (2009)



$nU = 1.2J$   
 $\alpha = 0.33 \pm 0.05$

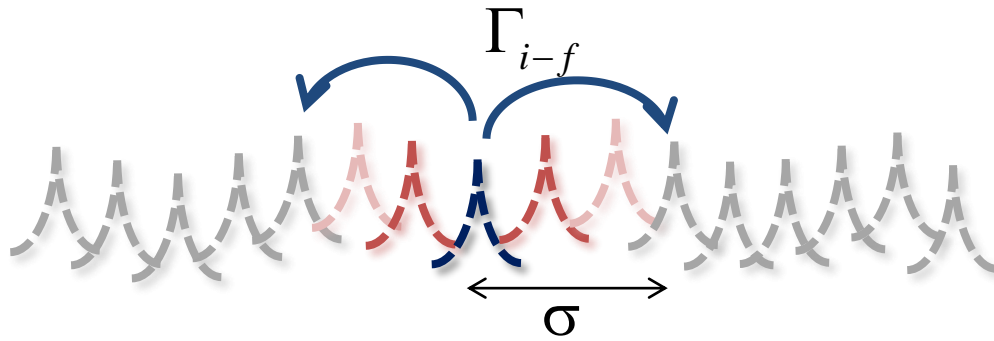
$nU = 0.8J$   
 $\alpha = 0.22 \pm 0.04$

$nU = 0$

$$\sigma(t) = \sigma_0 (1 + t/t_0)^\alpha$$



# Interaction-assisted transport



Perturbative approach:

$$H' = H_{\text{int}} = U \sum n_j (n_j - 1)$$

$$D \propto \Gamma_{i-f} \approx \frac{\langle i | H' | f \rangle^2}{\delta E} \propto n^\beta(x, t) \propto \frac{1}{\sigma^\beta}$$

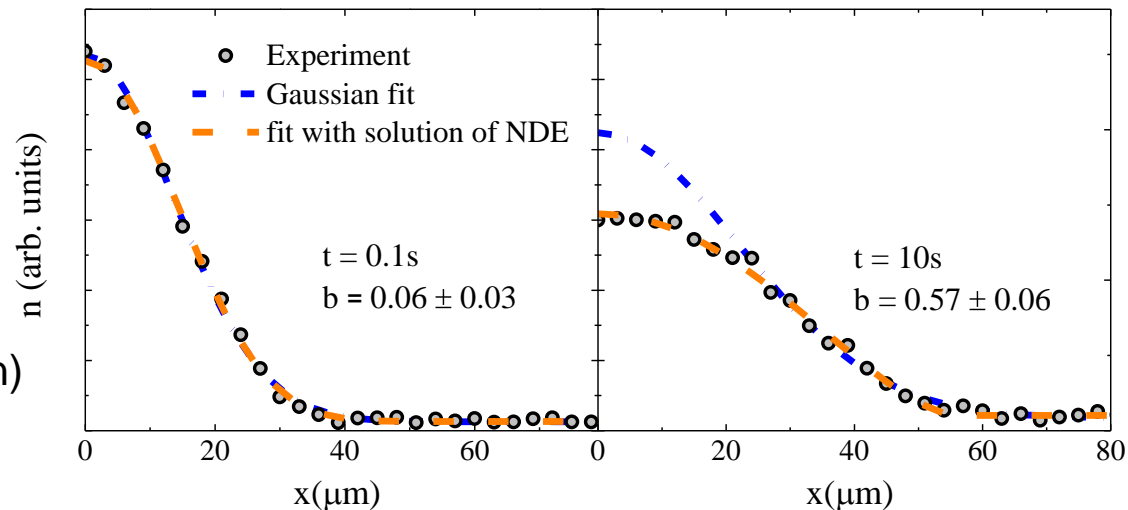
Non-linear diffusion equation:

$$\frac{\partial n(x, t)}{\partial t} = \frac{\partial}{\partial x} D(x, t) \frac{\partial n(x, t)}{\partial x}$$

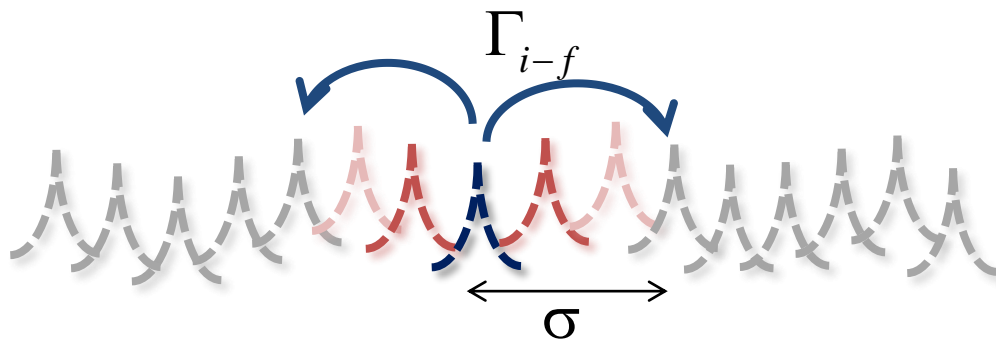
$$n(x, t) = B(b, w) \left( 1 - \frac{b(t)x^2}{w^2(t)} \right)^{1/b(t)}$$

$t=0$        $b(0) \rightarrow 0$  (Gaussian)  
 $t \gg 0$      $b(t) \rightarrow \beta$  (self-similar solution)

$$\sigma(t) \approx t^\alpha = t^{1/(2+\beta)}$$



# Noise-assisted transport



Perturbative approach:

$$H' = \Delta A \cos(\omega_i t) \cos(2\pi \frac{\lambda_1}{\lambda_2} x)$$

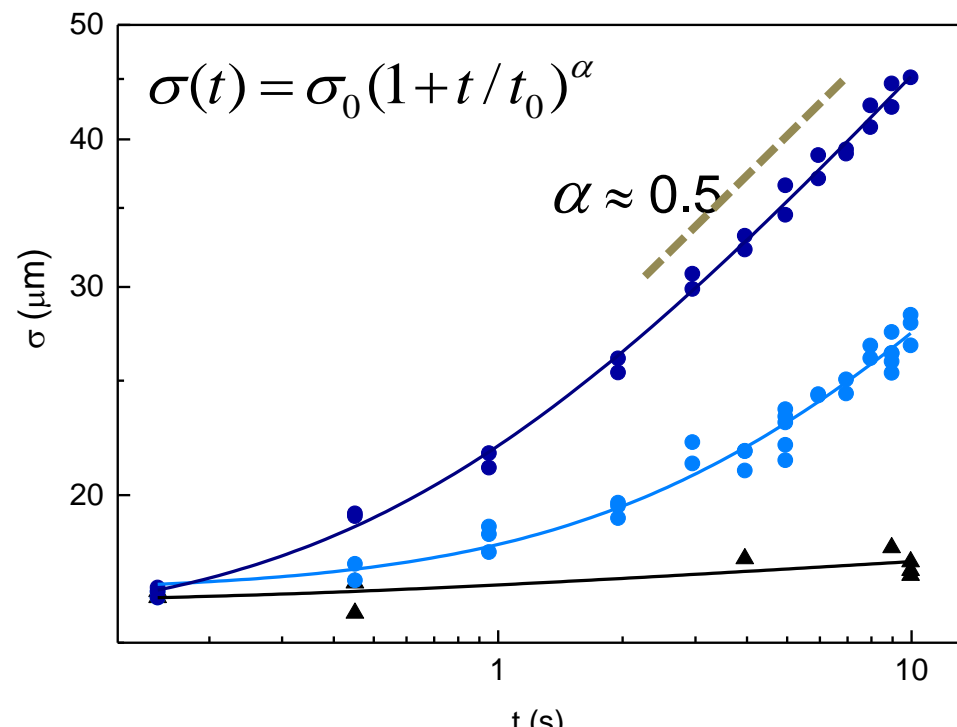
$$D \propto \Gamma_{i-f} \approx \frac{\langle i | H' | f \rangle^2}{\delta E} = \text{const}$$

Noise: sine modulation of the secondary lattice with a random frequency

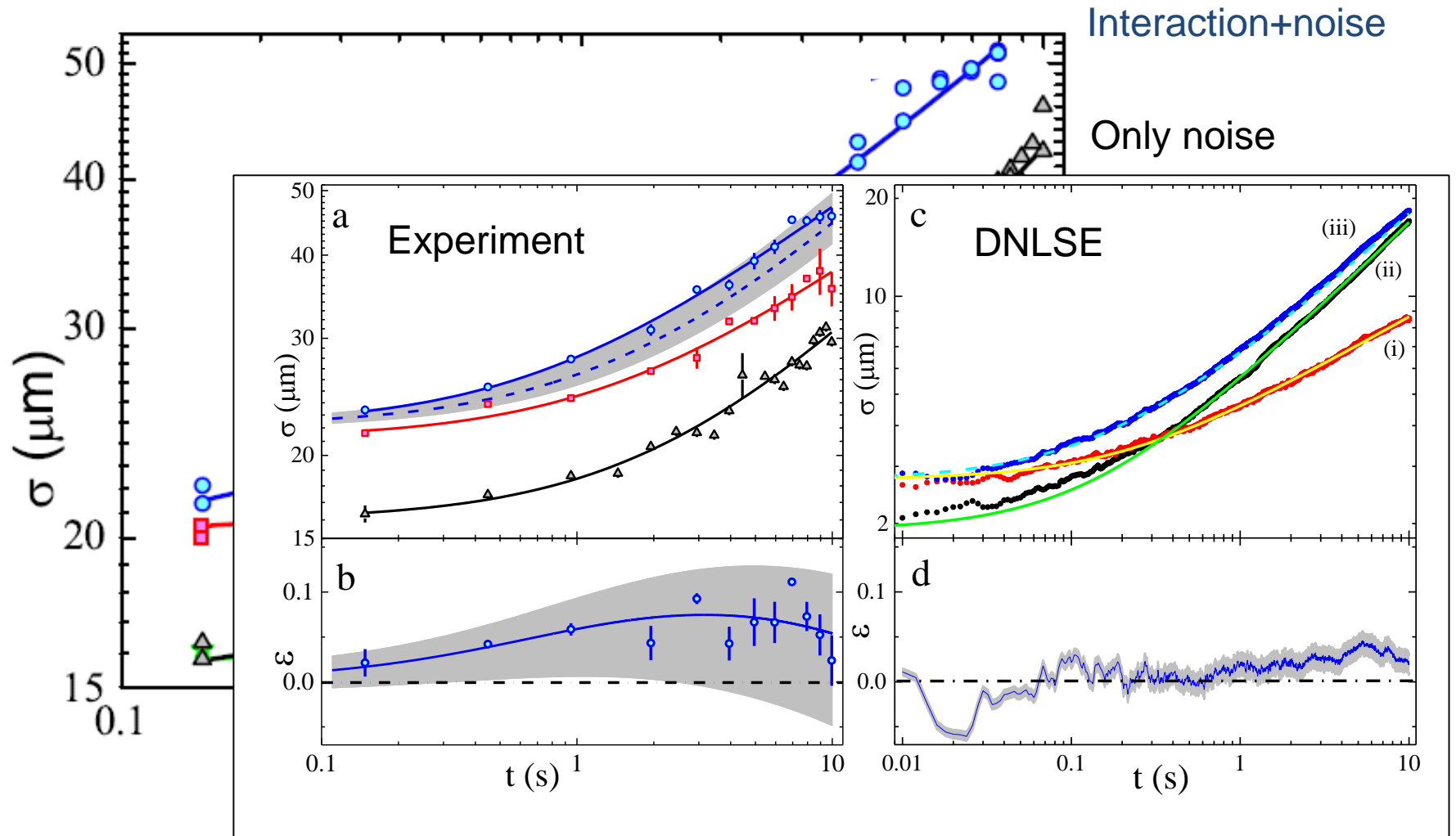
Standard diffusion equation:

$$\frac{\partial n(x,t)}{\partial t} = D \frac{\partial^2 n(x,t)}{\partial x^2}$$

$$\sigma(t) \approx t^{1/2}$$



# Interaction & Noise-assisted transport



$$D_{\text{int+noise}}(t) = D_{\text{noise}} + D_{\text{int}}(t)$$

# Experimental results and perspectives

- Investigation of the  $\Delta$ -U diagram and comparison with zero-temperature theory

C. D'Errico, E. Lucioni et al., submitted

*Reentrance of the Bose glass from coherence measurements*  
*Evidence of a strongly correlated BG from the excitation spectra*  
*Connection with zero-temperature phase diagram*

- Transport instability at the fluid-insulator transition:

L. Tanzi et al., Phys. Rev. Lett. (2013)

*Dominant role of quantum phase slips*  
*«experimental» transition: vanishing of critical momentum*

- Induced expansion dynamics in a disorder-localized system:

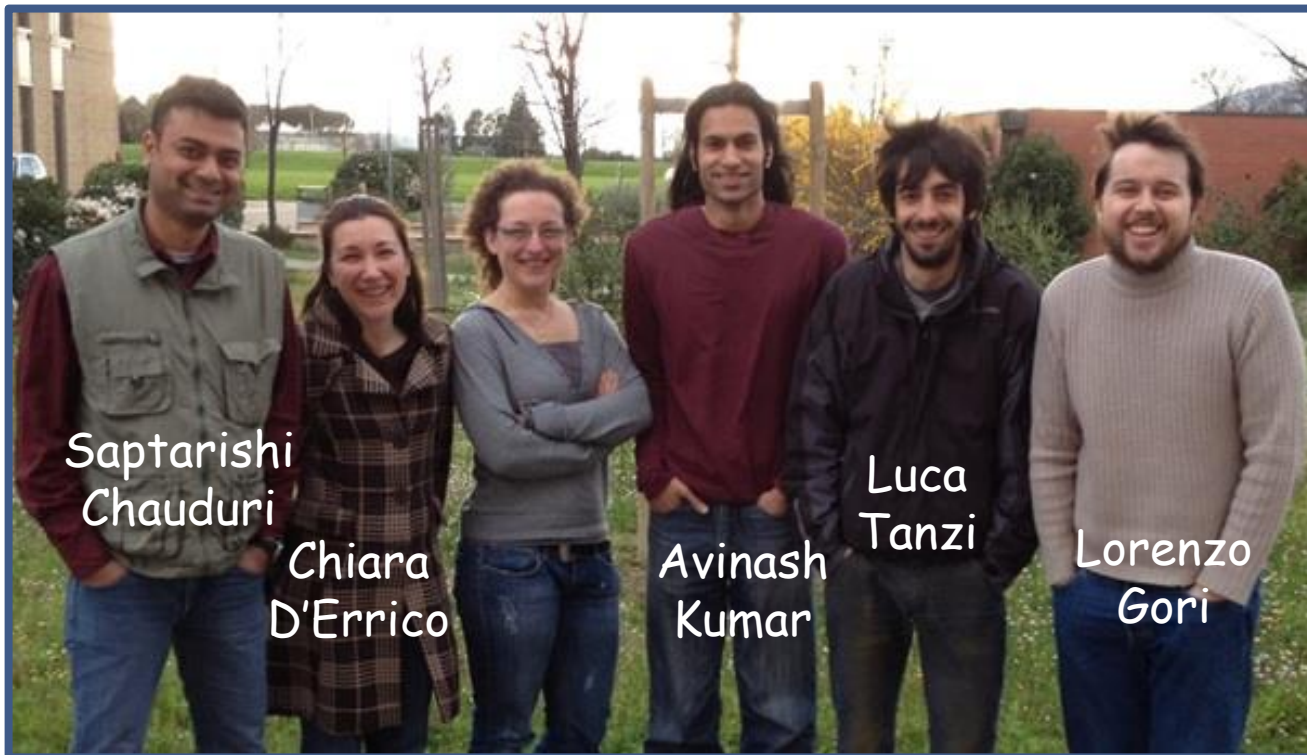
E. Lucioni et al., Phys. Rev. Lett. 106, 23 (2011),

E. Lucioni et al., Phys. Rev. E 87, 042922 (2013)

C. D'Errico et al., New J. Phys. 15, 045007 (2013)

*interaction-induced transport*  
*noise-induced transport*

- Perspectives: true disorder, higher dimensionality, role of the temperature, quantum quenches and thermalization in disordered system...



Saptarishi  
Chauduri

Chiara  
D'Errico

Avinash  
Kumar

Luca  
Tanzi

Lorenzo  
Gori



**Massimo  
Inguscio**  
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Marco Moratti

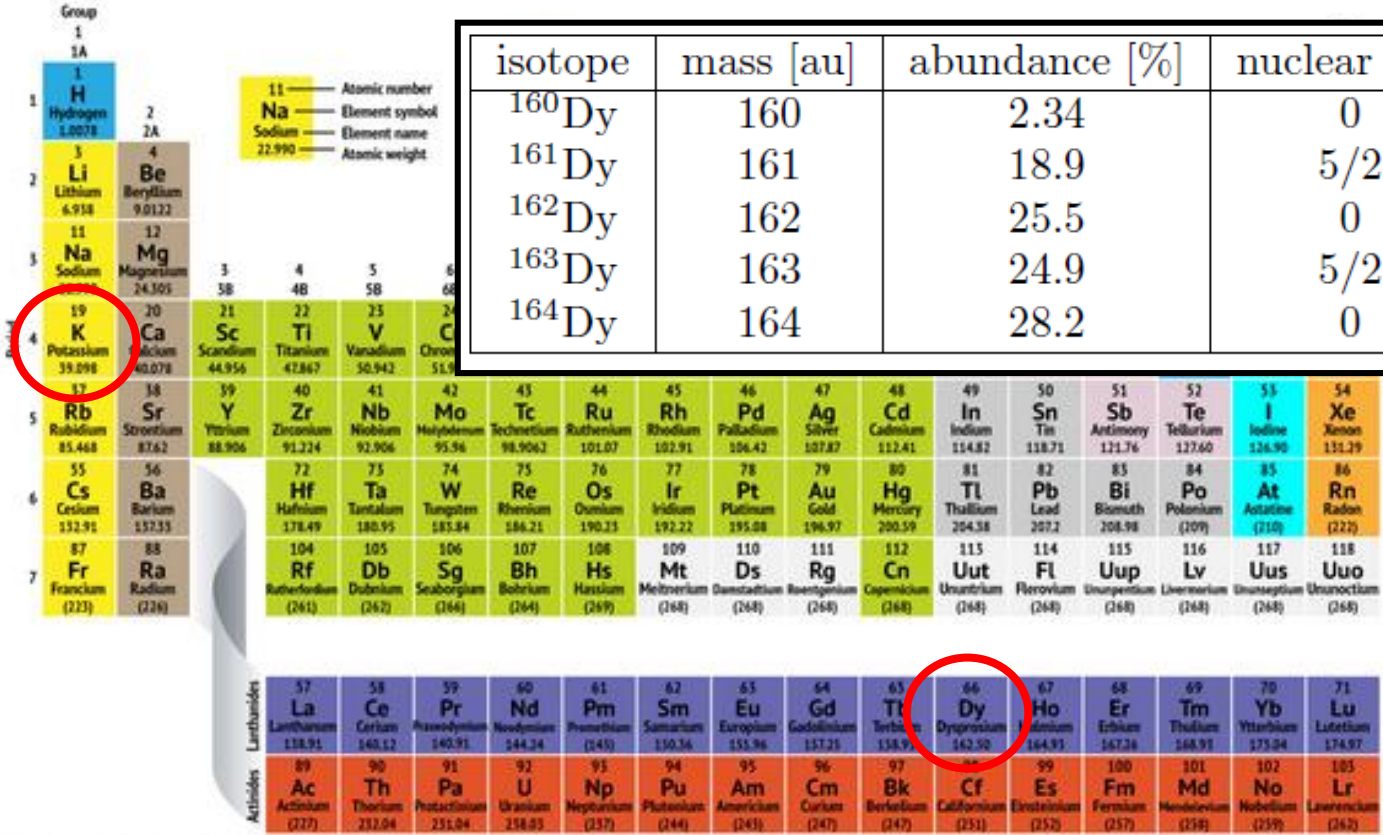
Filippo Caruso



*Università degli Studi di Firenze*

# Quantum gases of «new» atoms

## Periodic Table of the Elements



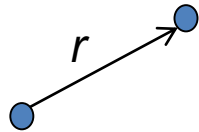
# Dipolar physics with Dysprosium

Contrary to alkaline atoms, Dysprosium has a magnetic dipole moment

$$v_c(r-r') = \frac{4\pi\hbar^2}{m} a \delta(r-r')$$

Contact interaction:

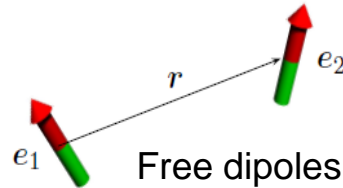
- Short range
- Isotropic



Tunable  
via Feshbach resonances

Dy is expected to have many resonances

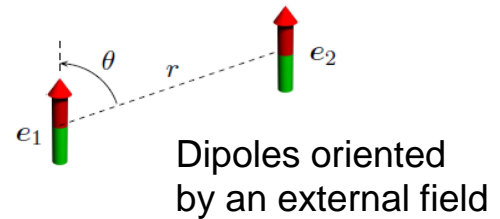
$$v_{dd}(r) = \frac{C_{dd}}{4\pi} \frac{(e_1 \cdot e_2)r^2 - 3(e_1 \cdot r)(e_2 \cdot r)}{r^5}$$



Dipole-dipole interaction:

- Long range
- Anisotropic

$$v_{dd}(r) = \frac{C_{dd}}{4\pi} \frac{1 - 3 \cos^2 \vartheta}{r^3}$$



$$C_{dd} = \begin{cases} \mu_0 \mu^2 \\ d^2 / \epsilon_0 \end{cases}$$

For Dy:  
 $\mu = 10\mu_B$

The highest  
In the periodic table

Dy PHYSICS IS DOMINATED  
BY DIPOLAR INTERACTION

Thank you  
for your attention!!!