Ground state stability of quantum dipolar filaments in BECs

Fabio Cinti

National Institute for Theoretical Physics
Stellenbosch, South Africa

Pisa, December 21, 2016

NITheP
National Institute for
Theoretical Physics

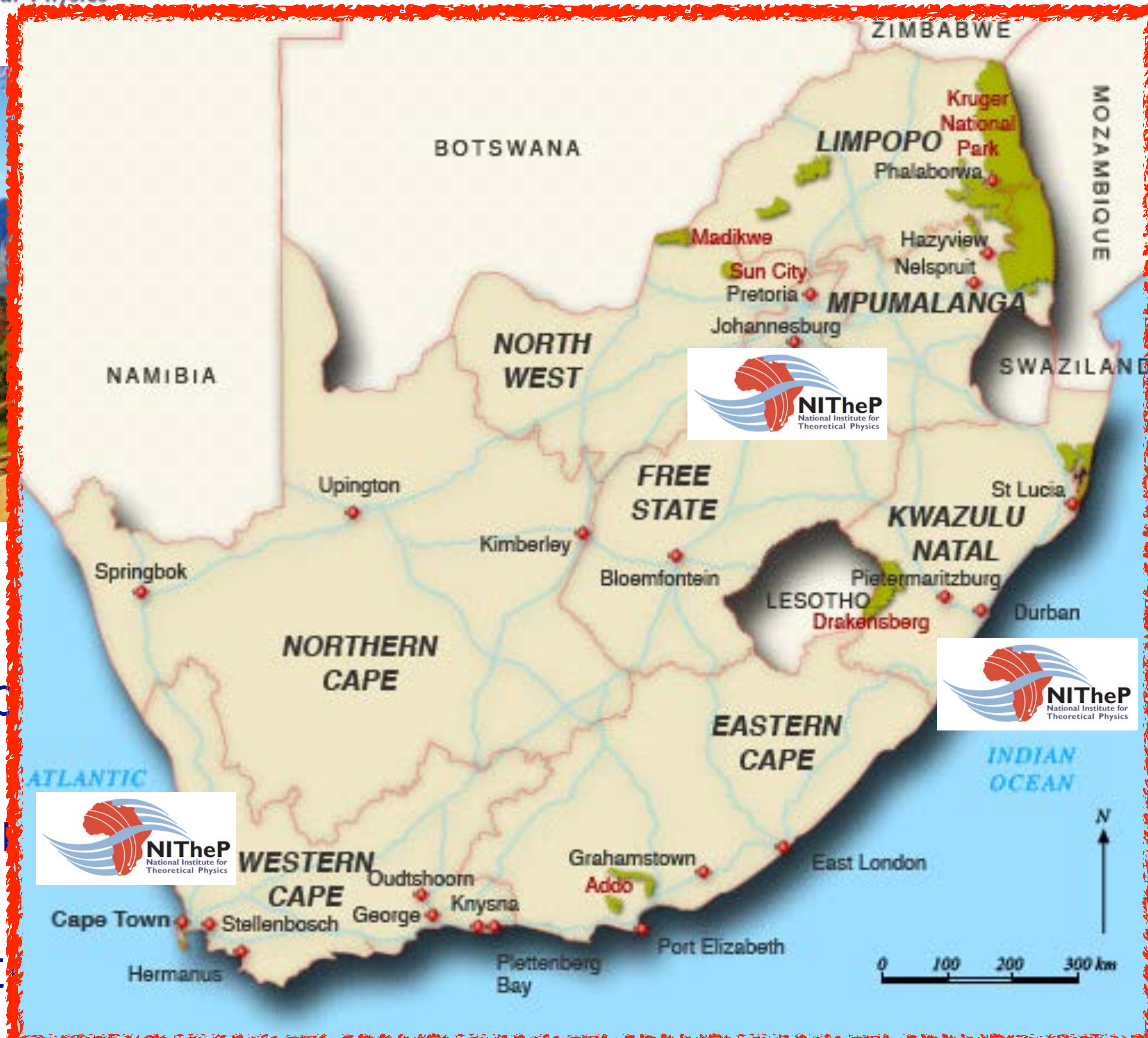
NITheP @ Stellenbosch

National Institute for Theoretical Physics

- Statistical Physics
- Quantum Field Theory
- Theoretical Condensed Matter
- Cold Atomic Quantum Gases

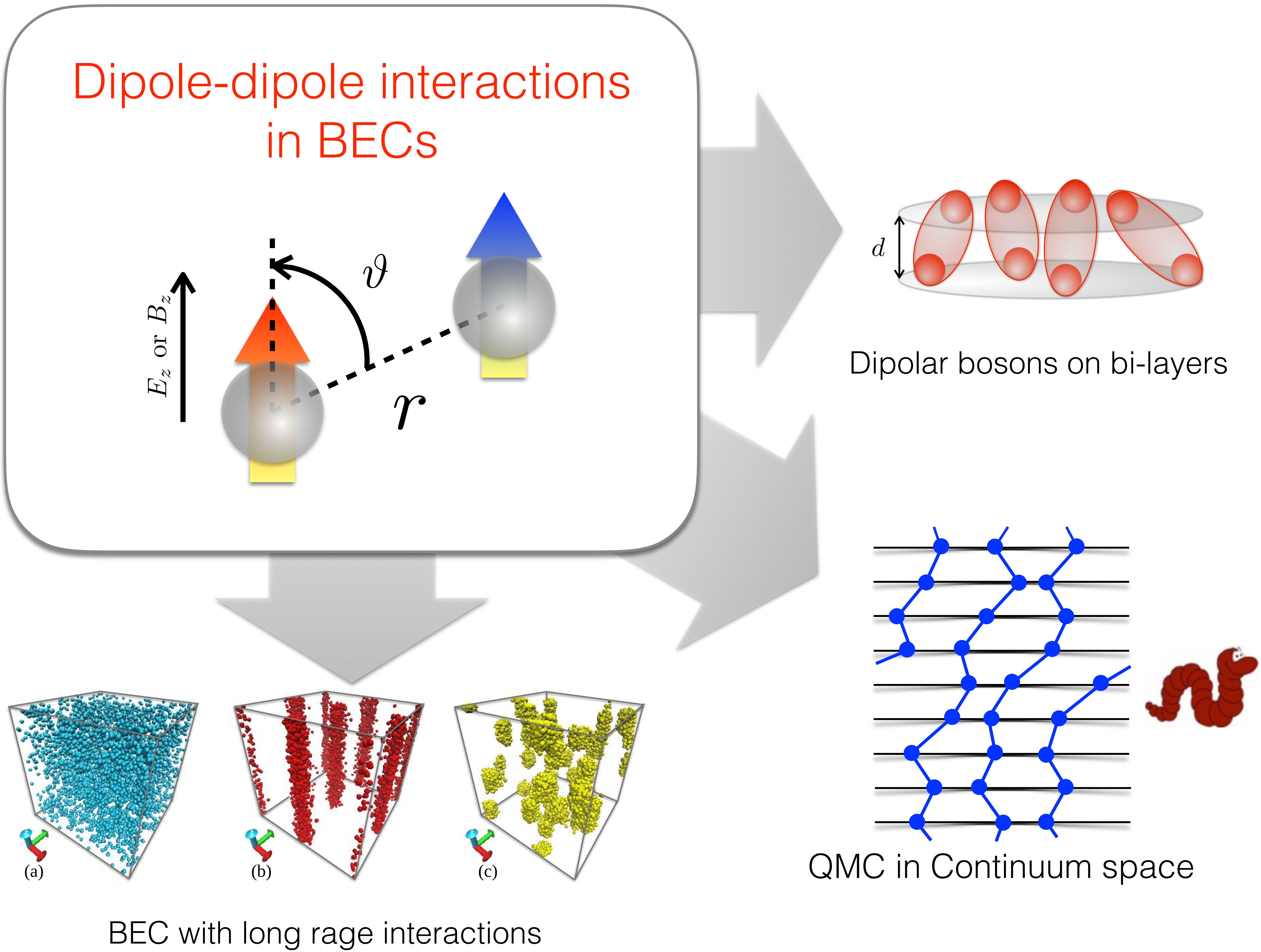
Visitor program
long-term visits!

NITheP @ Stellenbosch

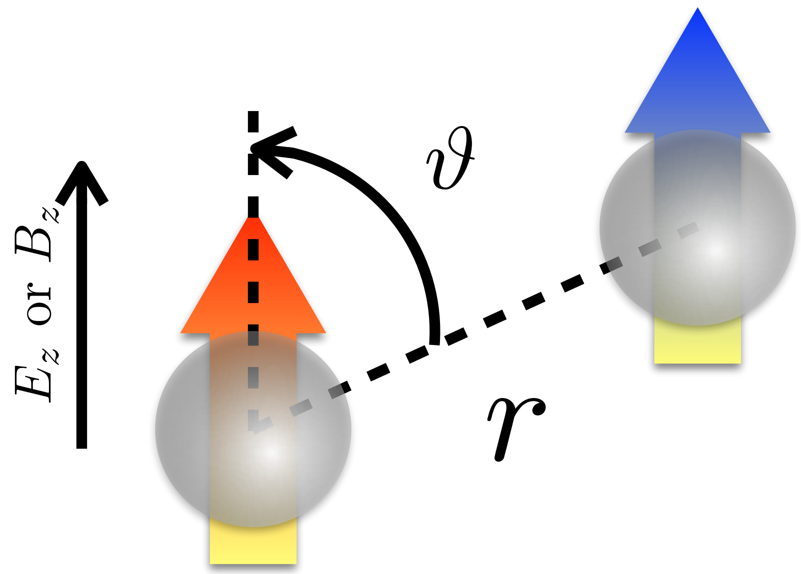


- Statistic
- Quantu
- Theoret
- Cold Atomic Quantum Gases

ogram
n visits!


Dipole-dipole interactions in BECs
E_z or B_z
ϑ
r
d
Dipolar bosons on bi-layers
QMC in Continuum space
(a)
(b)
(c)
BEC with long rage interactions

Dipole-dipole interactions



$$V(\mathbf{r}) = U(\mathbf{r}) + \frac{d^2}{r^3}\left(1 - 3\cos^2\theta\right)$$

d: **E** or **B** dipole moment

Two-body short-range interactions:

$$U(\mathbf{r}) = \frac{4\pi\hbar a}{m}\delta(\mathbf{r})$$

a s-wave scattering length

$a > 0$ repulsive

$a < 0$ attractive

$$g = \frac{4\pi\hbar^2 a(d)}{m}$$

$$a_d = \frac{md^2}{\hbar^2}$$

Characteristic range of the interactions

Ultra-cold atoms

Typical temperatures

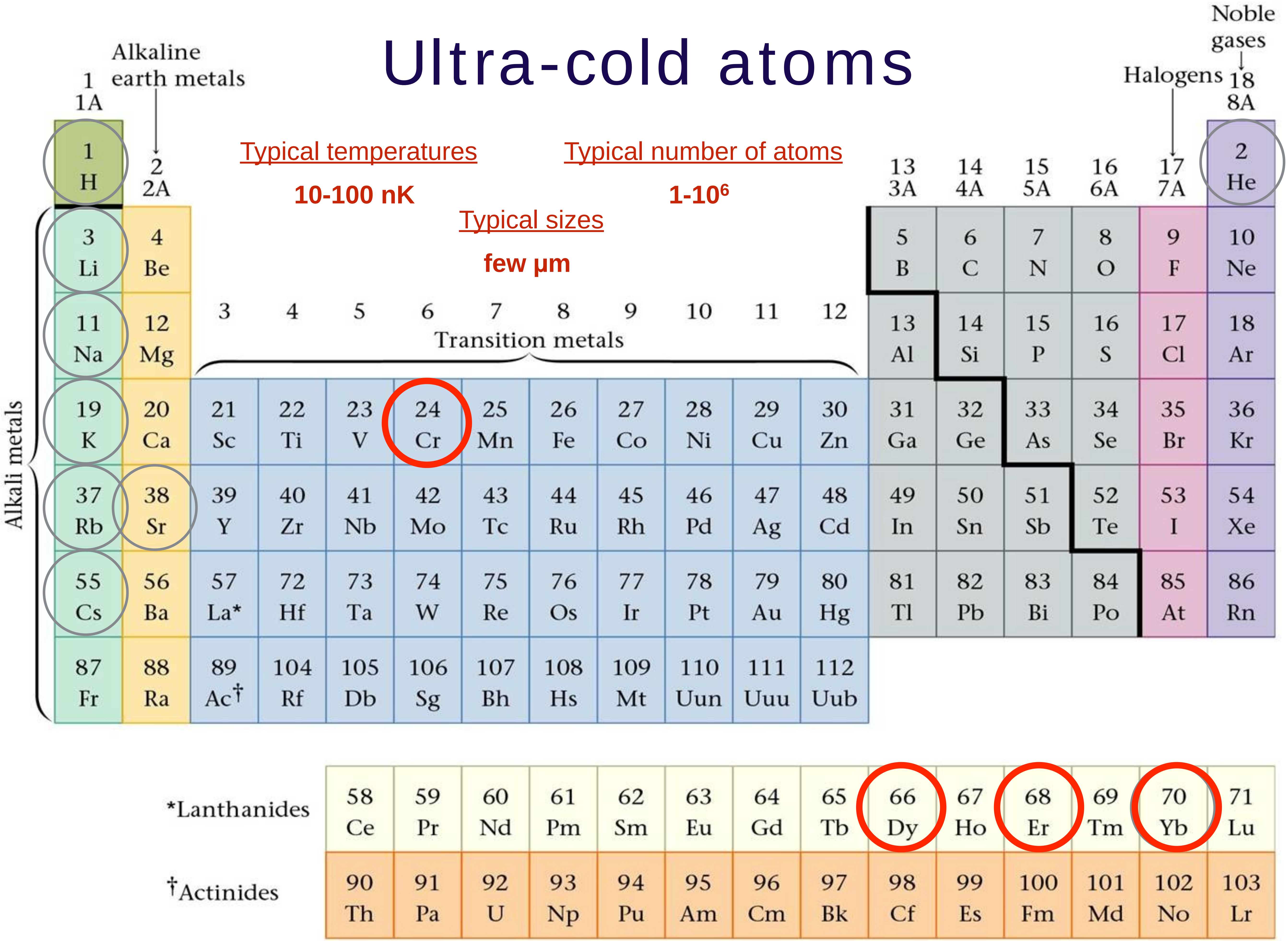
10-100 nK

Typical number of atoms

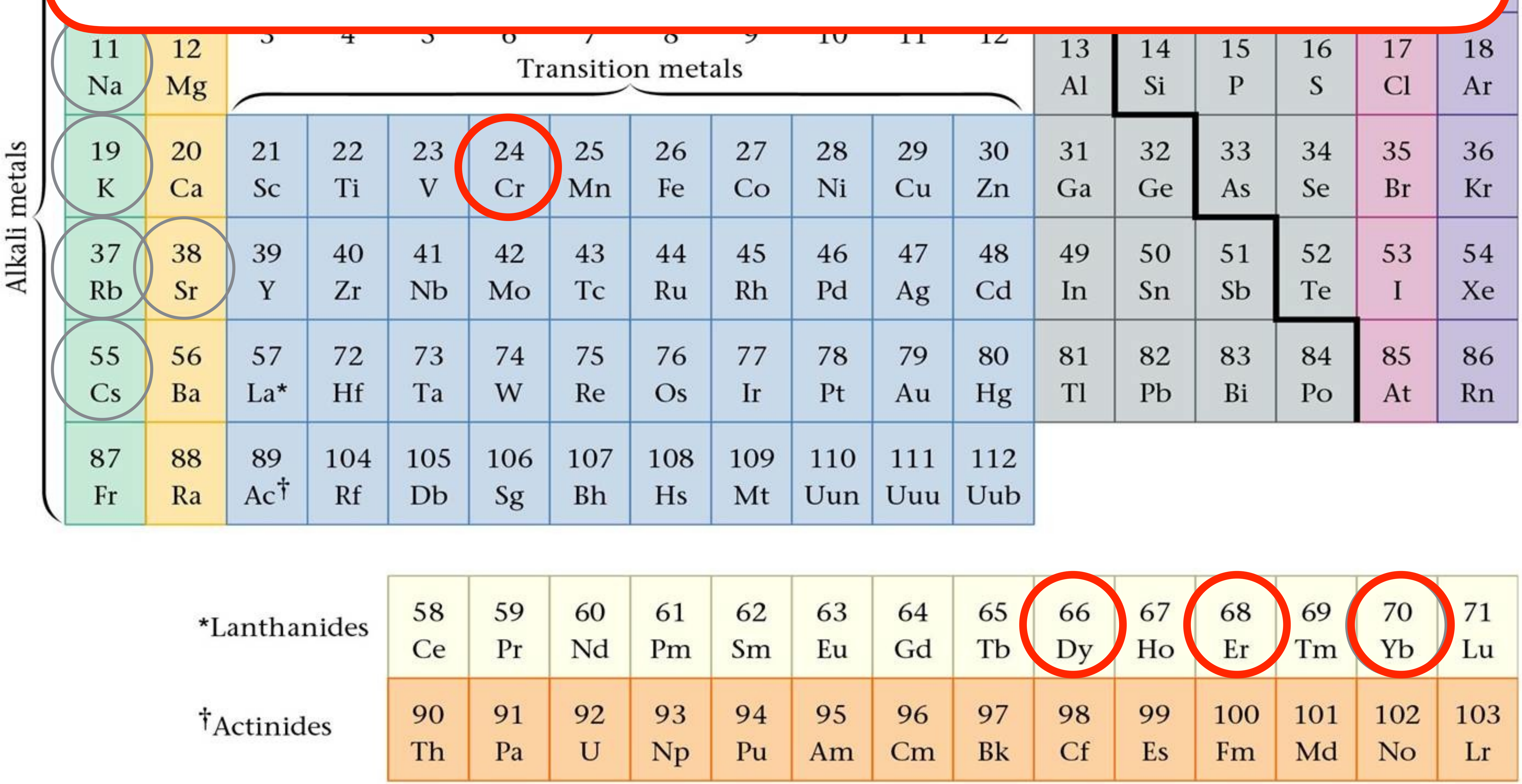
1-10^6

Typical sizes

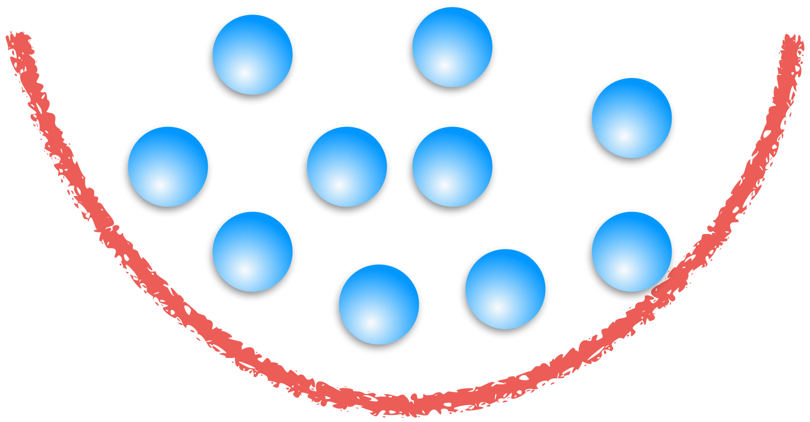
few µm



- Cr (d=6μ_B), Er (7μ_B), Dy (10μ_B)
- Polar molecules (KRb): $d\approx 0.1 \div 1$D
- Tunability of the dipole-dipole interactions (Feshbach resonance)!

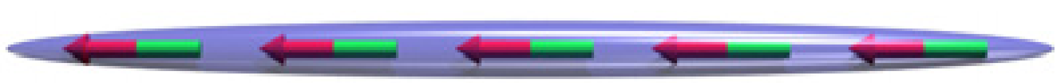


Dipole-dipole interactions



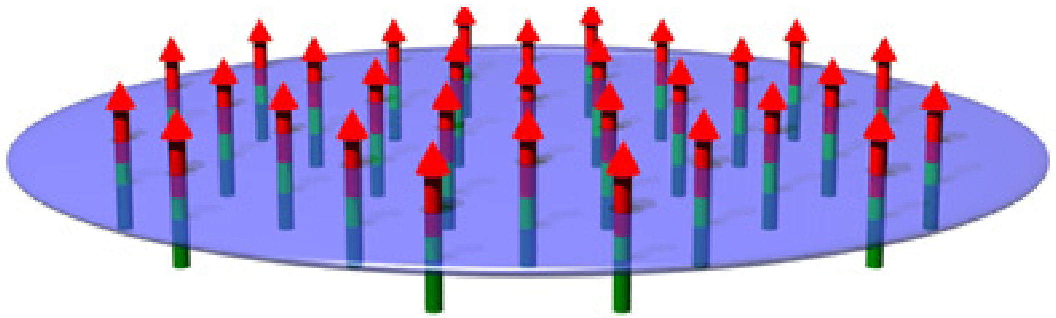
3D harmonic trapping

$$V_{\text{ext}}(\mathbf{r}) = \frac{m}{2}\left[\omega_\rho^2\left(x^2+y^2\right)+\omega_z^2 z^2\right] \qquad \lambda = \frac{\omega_z}{\omega_\rho}$$



Attraction (unstable)

$$\lambda > 1$$



Repulsion (stable)

$$\lambda < 1$$

Lahaye et al. 2009
Baranov et al. 2012

Cr

LETTERS
nature physics | VOL 4 | MARCH 2008 | www.nature.com/naturephysics

Stabilization of a purely dipolar quantum gas against collapse

T. KOCH, T. LAHAYE, J. METZ, B. FRÖHLICH, A. GRIESMAIER AND T. PFAU*
5. Physikalisches Institut, Universität Stuttgart, Pfaffenwaldring 57, 70550 Stuttgart, Germany
*e-mail: t.pfau@physik.uni-stuttgart.de

$$\varepsilon_{dd} = \frac{1}{3}\frac{a_d}{a}$$

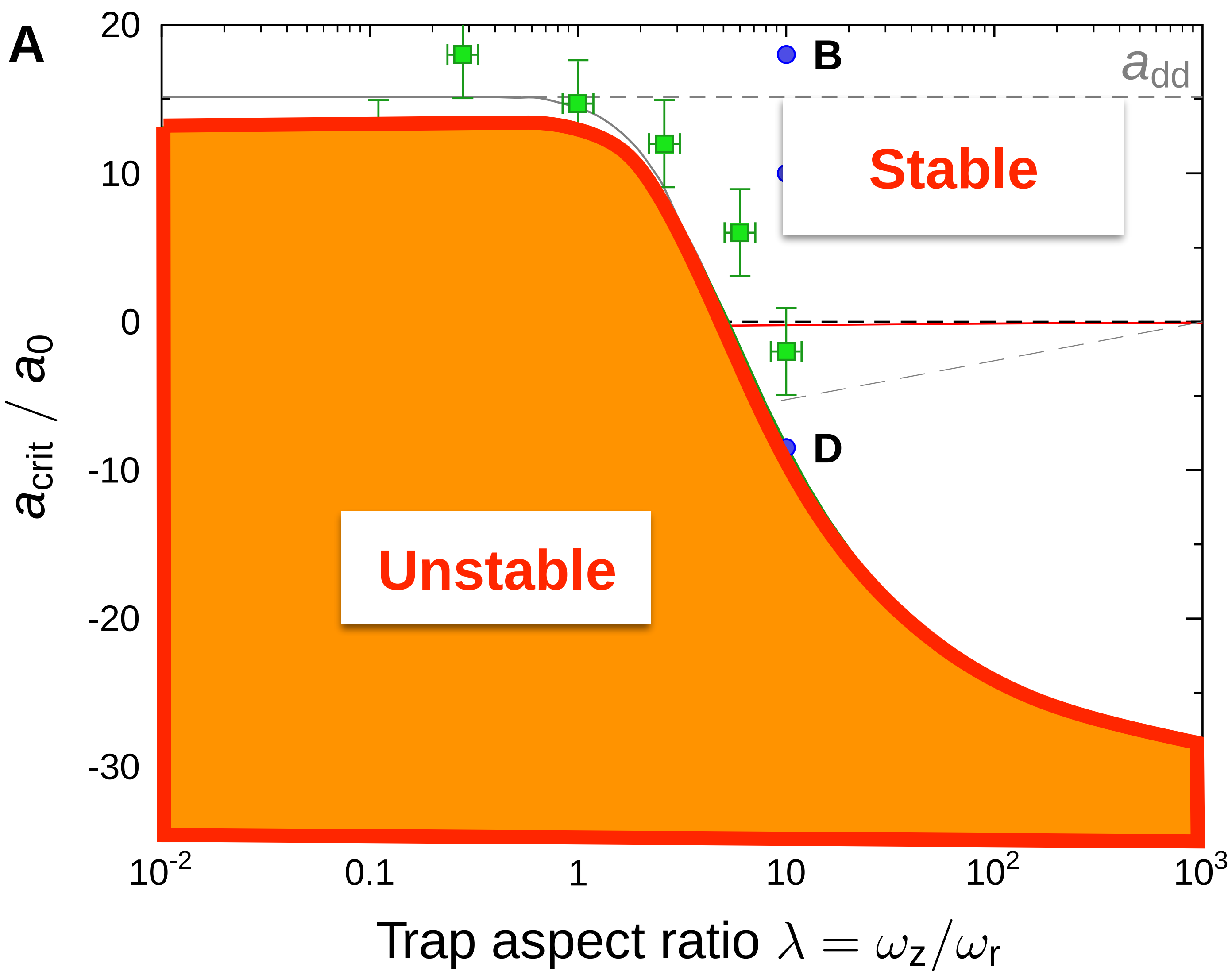
- short-range are dominant dipole–dipole only small corrections

$$\varepsilon_{dd} < 1$$

- dipole–dipole are dominant condensate instability

$$\varepsilon_{dd} > 1$$

BECs stability is trap-dependent



LETTER

Nature **530**, 194–197 (11 February 2016)

Observing the Rosensweig instability of a quantum ferrofluid

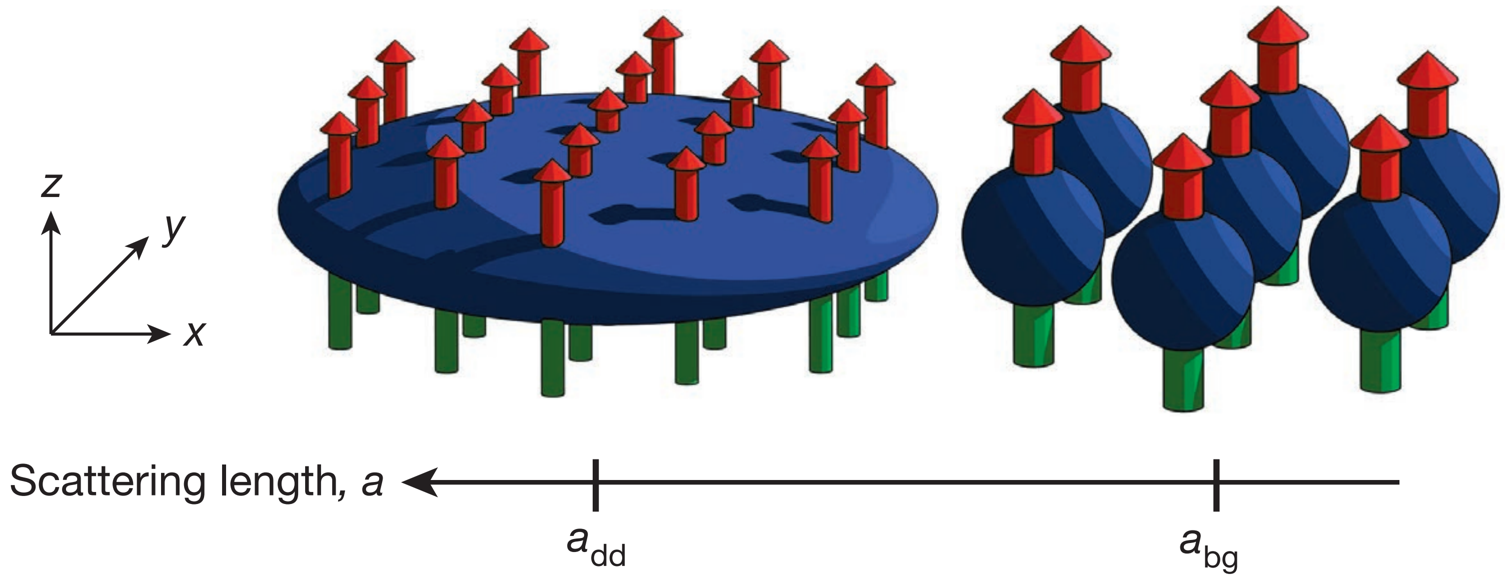
Holger Kadau[1], Matthias Schmitt[1], Matthias Wenzel[1], Clarissa Wink[1], Thomas Maier[1], Igor Ferrier-Barbut[1] & Tilman Pfau[1]

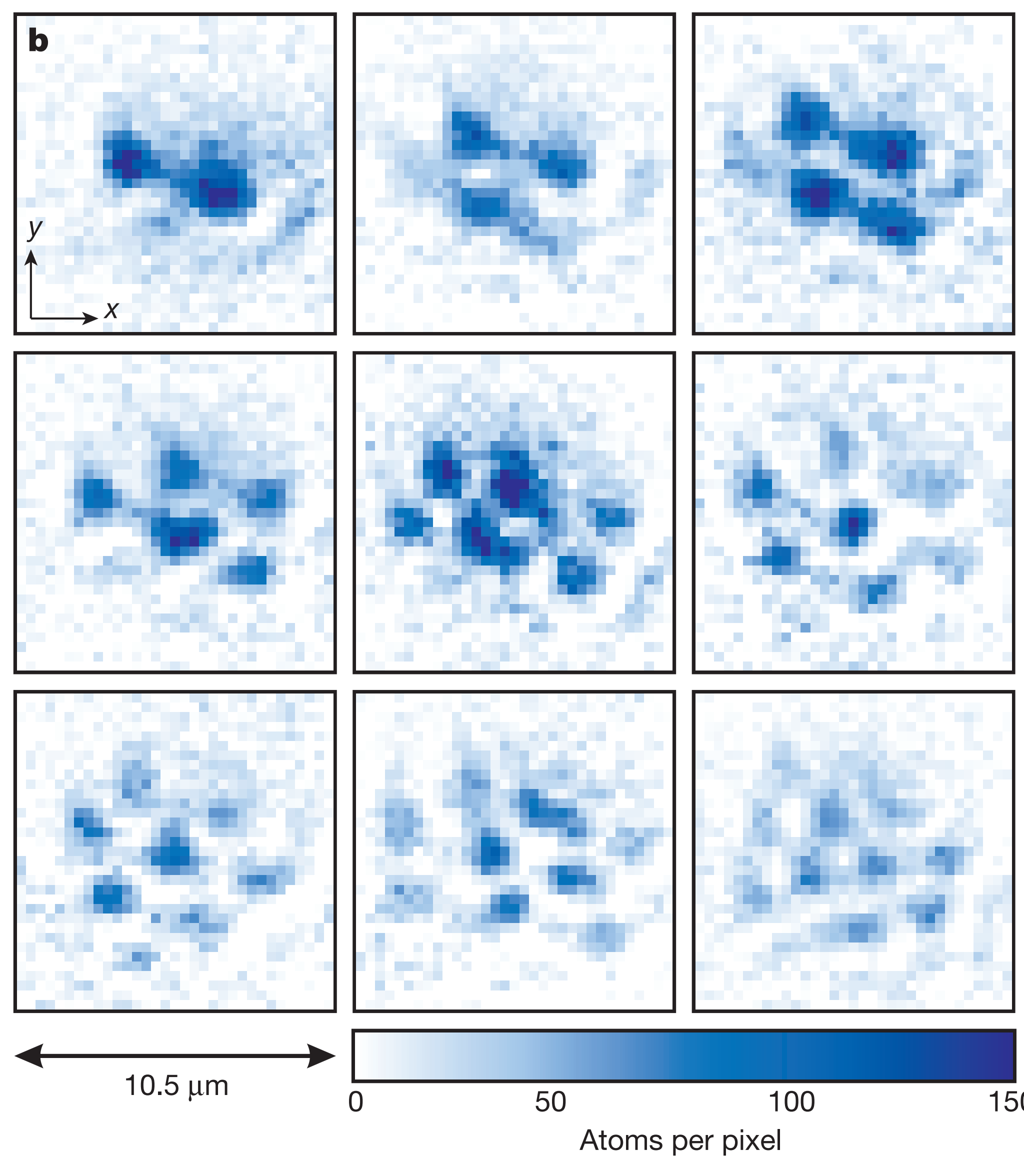
Ferrofluids exhibit unusual hydrodynamic effects owing to the magnetic nature of their constituents. As magnetization increases, a classical ferrofluid undergoes a Rosensweig instability[1] and creates self-organized, ordered surface structures[2] or droplet crystals[3]. Quantum ferrofluids such as Bose–Einstein condensates with strong dipolar interactions also display superfluidity[4]. The field of dipolar quantum gases is motivated by the search for new phases of matter that break continuous symmetries[5,6]. The simultaneous breaking of continuous symmetries such as the phase invariance in a superfluid state and the translational symmetry in a crystal provides the basis for these new states of matter. However, interaction-induced crystallization in a superfluid has not yet been observed. Here we use *in situ* imaging to directly observe the spontaneous transition from an unstructured superfluid to an ordered arrangement of droplets in an atomic dysprosium Bose–Einstein condensate[7]. By using a Feshbach resonance to control the interparticle interactions, we induce a finite-wavelength instability[8] and observe discrete droplets in a triangular structure, the number of which grows as the number of atoms increases. We find that these structured states are surprisingly long-lived and observe hysteretic behaviour, which is typical for a crystallization process and in close analogy to the Rosensweig instability. Our system exhibits both superfluidity and, as we show here, spontaneous translational symmetry breaking. Although our observations do not probe superfluidity in the structured states, if the droplets establish a common phase via weak links, then our system is a very good candidate for a supersolid ground state[9–11].

interaction in a quantum ferrofluid. For increasing relative dipolar interaction, the roton instability can lead to a periodic perturbation of the atomic density distribution, which is closely connected to the Rosensweig instability[17]. However, it was believed that these rotonic structures would be unstable, owing to subsequent instabilities of the forming droplets[18].

Here we cool down the most magnetic element—dysprosium (Dy)[19], with a magnetic moment of $\mu = 9.93\mu_B$, where μ_B is the Bohr magneton—and generate a Bose–Einstein condensate (BEC)[7]. We observe an angular roton instability[16,18] and find subsequent droplet formation yielding triangular structures with surprisingly long lifetimes. We use two key tools to study these self-organized structures. First, we use a magnetic Feshbach resonance[20] to tune the contact interaction (see Extended Data Fig. 1) and to induce the droplet formation. Second, we use a microscope with high spatial resolution to detect the atomic density distribution *in situ*.

The first prediction of structured ground states in a dipolar BEC dates back to the early days of quantum gases[21]; the first mechanical effects were seen with chromium atoms[22]. There, the dipolar attraction deforms the compressible gas and its shape is balanced by a repulsive contact interaction, described by the scattering length a. To compare the strengths of the contact and dipolar interaction, we introduce a length scale that characterizes the magnetic dipole–dipole interaction strength: $a_{dd} = \mu_0 \mu^2 m/(12\pi\hbar)$ (ref. 2), where μ_0 is the vacuum permeability, m is the atomic mass and $\hbar$ is the reduced Planck constant. By tuning the scattering length a with a Feshbach resonance such that $a < a_{dd}$, the dipolar attraction dominates the repulsive contact inter-


z
y
x
Scattering length, a
a_{dd}
a_{bg}


b
y
x
10.5 μm
0
50
100
150
Atoms per pixel

$$a_d > a$$

PHYSICAL REVIEW X **6,** 041039 (2016)

Er

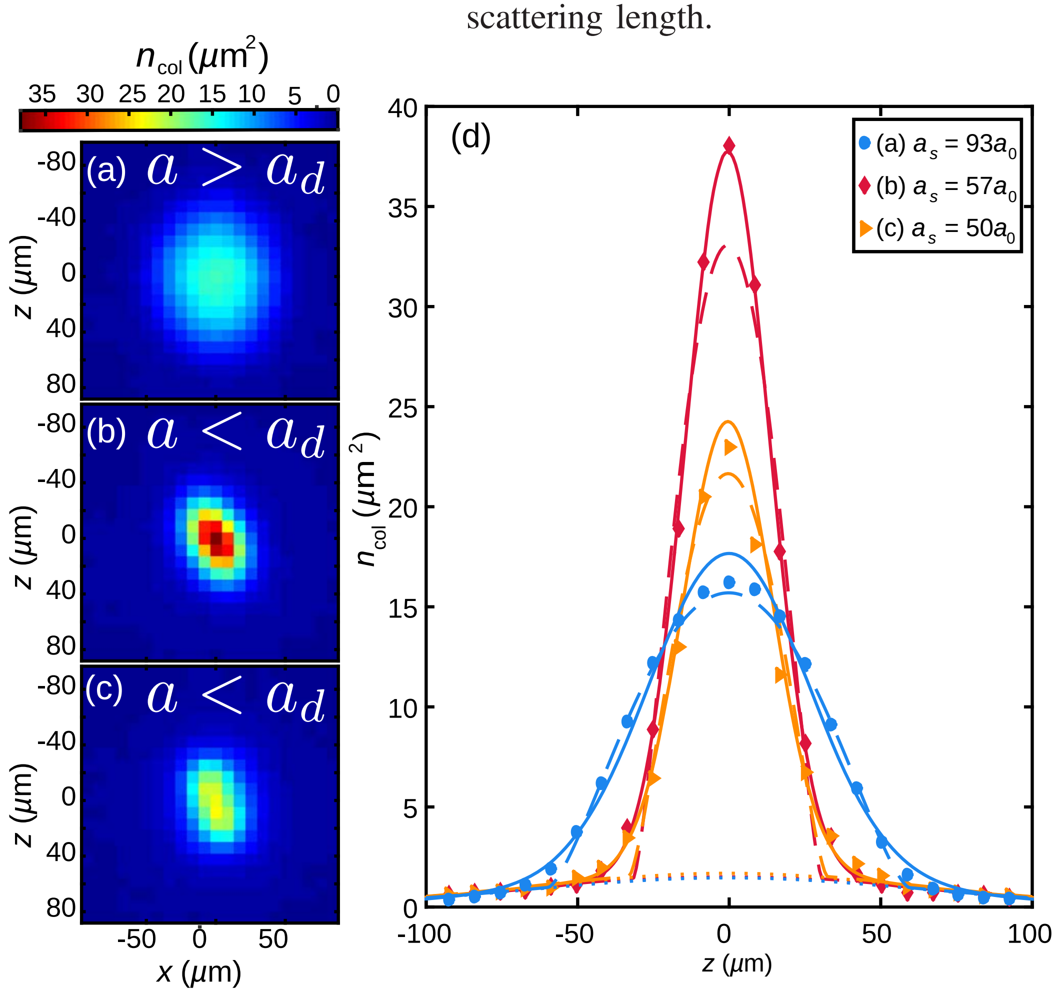
Quantum-Fluctuation-Driven Crossover from a Dilute Bose-Einstein Condensate to a Macrodroplet in a Dipolar Quantum Fluid

L. Chomaz,[1] S. Baier,[1] D. Petter,[1] M. J. Mark,[1,2] F. Wächtler,[3] L. Santos,[3] and F. Ferlaino[1,2,*]

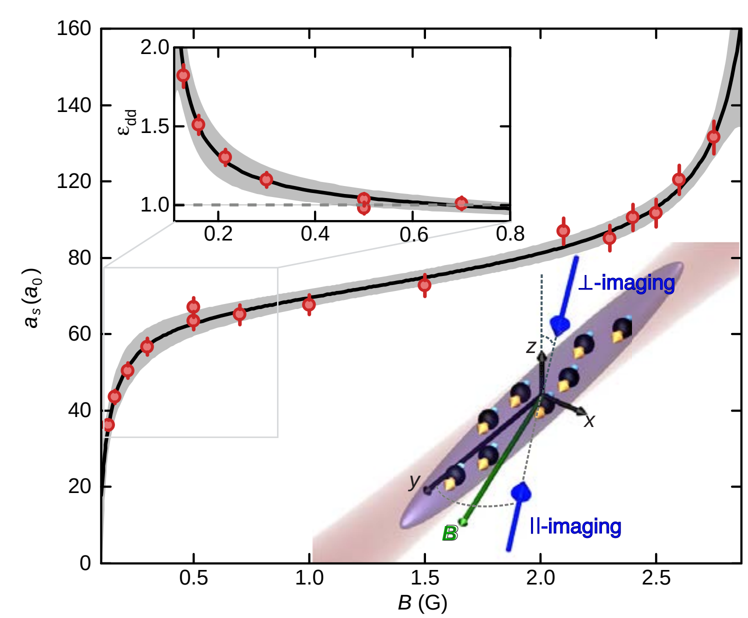
[1]*Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria*
[2]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*
[3]*Institut für Theoretische Physik, Leibniz Universität Hannover, Appelstrasse 2, 30167 Hannover, Germany*

(Received 22 July 2016; revised manuscript received 15 September 2016; published 22 November 2016)

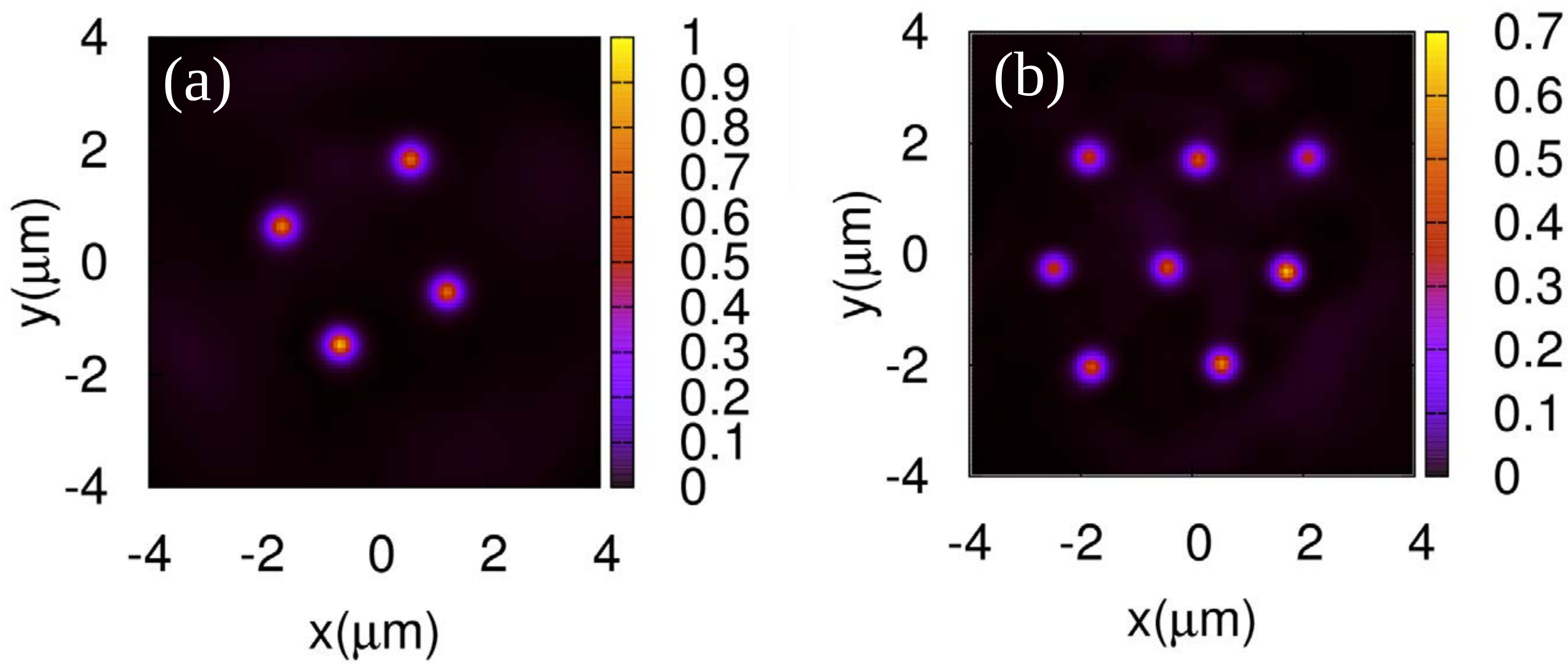
In a joint experimental and theoretical effort, we report on the formation of a macrodroplet state in an ultracold bosonic gas of erbium atoms with strong dipolar interactions. By precise tuning of the s-wave scattering length below the so-called dipolar length, we observe a smooth crossover of the ground state from a dilute Bose-Einstein condensate to a dense macrodroplet state of more than 2×10^4 atoms. Based on the study of collective excitations and loss features, we prove that quantum fluctuations stabilize the ultracold gas far beyond the instability threshold imposed by mean-field interactions. Finally, we perform expansion measurements, showing that although self-bound solutions are prevented by losses, the interplay between quantum stabilization and losses results in a minimal time-of-flight expansion velocity at a finite scattering length.



up to $\varepsilon_{dd} \approx 1.3$



Dipolar Bose-Einstein condensate: quantum filaments



$$i\hbar\dot{\psi}(\mathbf{r}) = \left[\hat{H}_0 + \mu_0(n(\mathbf{r}), \epsilon_{dd}) + \Delta\mu(n(\mathbf{r}), \epsilon_{dd})\right]\psi(\mathbf{r})$$

- Non-linear non-local Schroedinger equation (modified Gross-Pitaevskii equation)
- Stability due to Lee-Huang-Yang (LHY) corrections
- LHY has a repulsive correction (~$n^{3/2}$)
- No Three-Body losses

Waechtler and Santos 2016

see also Baillie et al. 2016

Dipolar Bose-Einstein condensate: quantum filaments

Beyond Mean-field approaches applied so far: QMC

- Sampling fundamental properties like condensation and superfluidity at finite temperature.
- High density & and strong interaction regimes
- Studying of quantum many-body phases

and Santos 2016

- No Three-Body losses

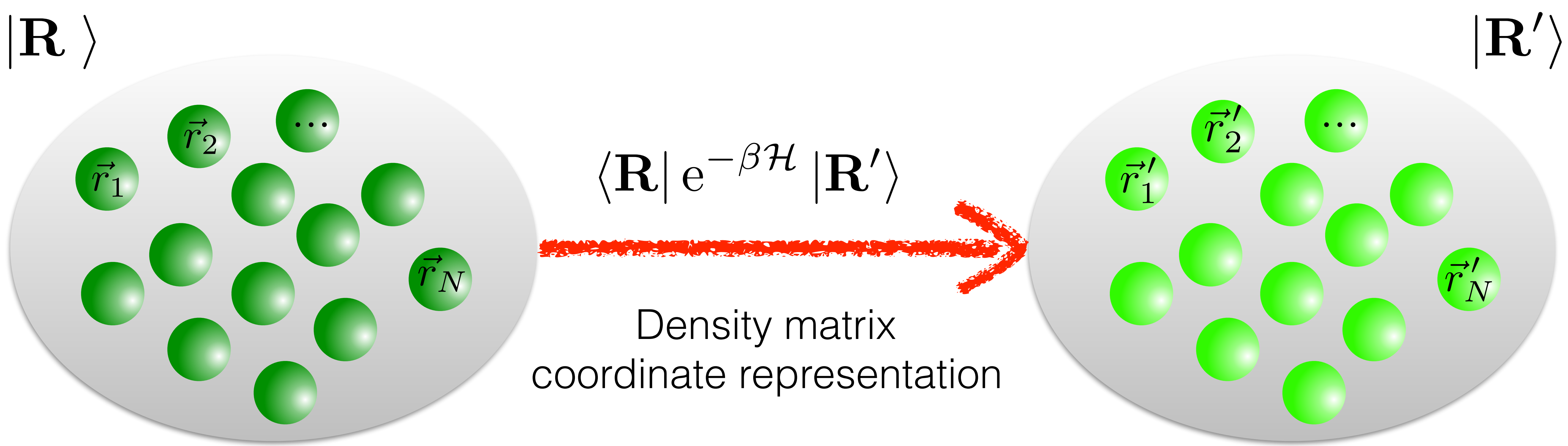
see also Baillie et al. 2016

Many-body approach: density matrix

$$\langle \mathcal{O} \rangle = \frac{\mathrm{Tr}\, \mathcal{O}\, \mathrm{e}^{-\beta \mathcal{H}}}{\mathrm{Tr}\, \mathrm{e}^{-\beta \mathcal{H}}}$$

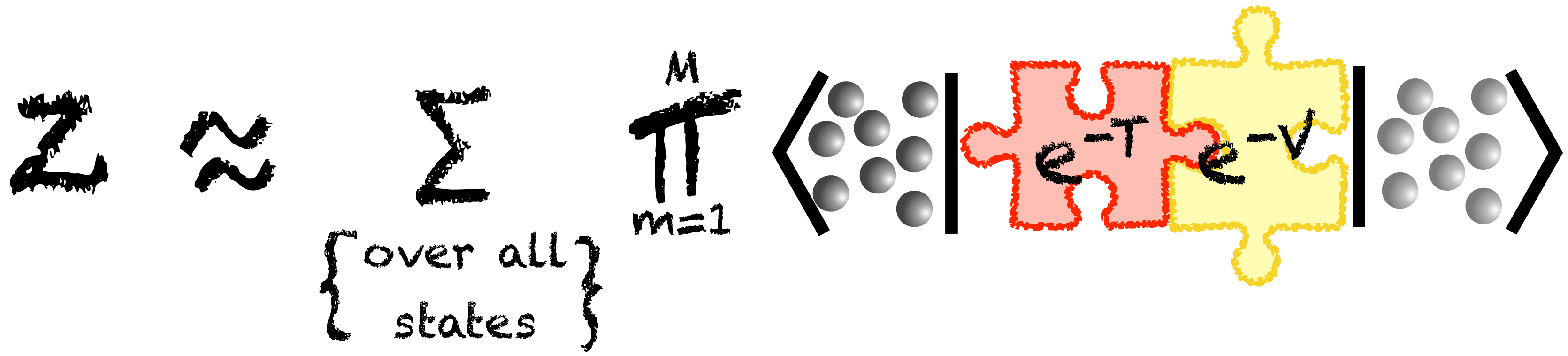
$$\beta = \frac{1}{k_B T}$$

- Energy
- Superfluidity
- Condensation
- Correlations & dynamical properties
- Structural properties
- …



Ceperley RMP 1995

Partition function



Partition function

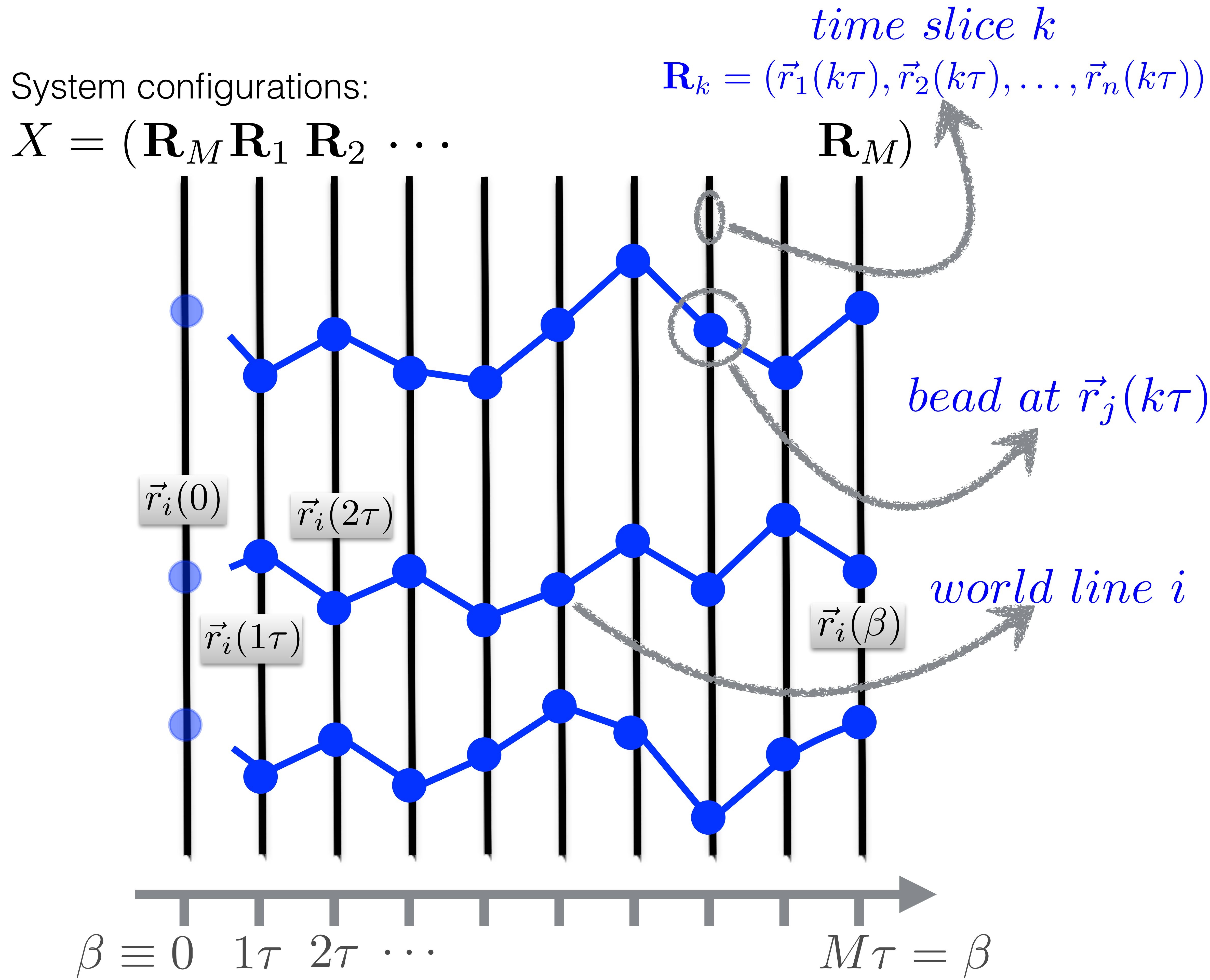
$$Z(N,V,T) = \int d\mathbf{R}\, \rho(\mathbf{R},\mathbf{R},\beta)$$

$$= \int \cdots \int \prod_{m=1}^{M} d\mathbf{R}_m\, \rho^{\text{free}}\left(\mathbf{R}_m, \mathbf{R}_{m+1}, \tau\right) e^{-\tau U(\mathbf{R}_m, \mathbf{R}_{m+1})}$$

- PF of a classical system of polymers. Every polymer is a necklace of beads connected by springs

- The mean square displacement of the polymer's beads is of the order of the de Broglie thermal wave length $\lambda_{dB} = \sqrt{4\pi\lambda\beta}$

- Famous mapping from quantum to classical system proposed by Feynman (see superfluidity in ^{4}He)



time slice k
$\mathbf{R}_k = (\vec{r}_1(k\tau), \vec{r}_2(k\tau), \ldots, \vec{r}_n(k\tau))$
System configurations:
$X = (\mathbf{R}_M \mathbf{R}_1 \; \mathbf{R}_2 \; \cdots \qquad \mathbf{R}_M)$
bead at $\vec{r}_j(k\tau)$
$\vec{r}_i(0)$
$\vec{r}_i(2\tau)$
world line i
$\vec{r}_i(1\tau)$
$\vec{r}_i(\beta)$
$\beta \equiv 0 \quad 1\tau \quad 2\tau \quad \cdots$
$M\tau = \beta$

time slice k

$\mathbf{R}_k = (\vec{r}_1(k\tau), \vec{r}_2(k\tau), \ldots, \vec{r}_n(k\tau))$

System configurations:

$X = (\mathbf{R}_M \mathbf{R}_1 \mathbf{R}_2 \cdots \mathbf{R}_M)$

bead at $\vec{r}_j(k\tau)$

world line i

Strategy

- Efficiently evaluation of integrals in *dNM* dimension.
- Stochastic non-uniform sampling (importance sampling)
- Statistical errors:

$$\Delta_{\mathcal{O}} \propto \sqrt{\frac{\mathrm{Var}(\mathcal{O})}{\#\mathrm{measure}}}$$

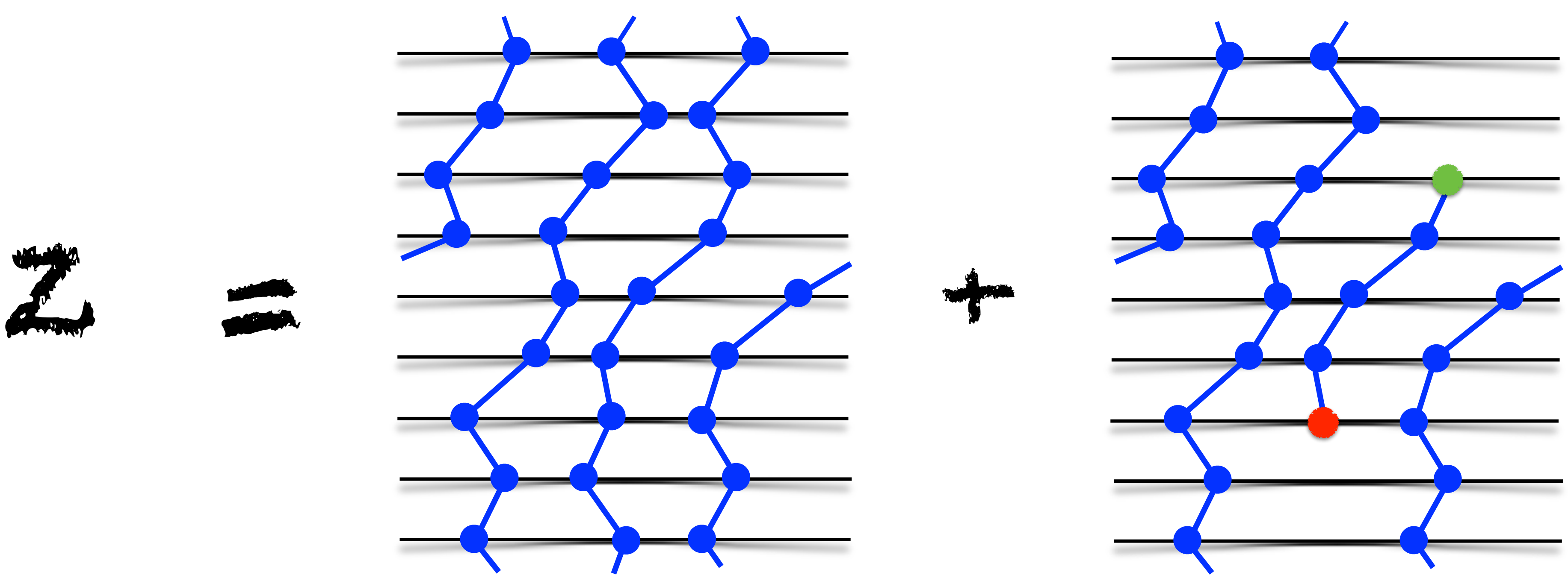
$\beta = 0 \quad 1\tau \quad 2\tau \quad \cdots \quad M\tau = \beta$

Worm algorithm

The configuration space is generalised from the partition function to the Matsubara-Green function

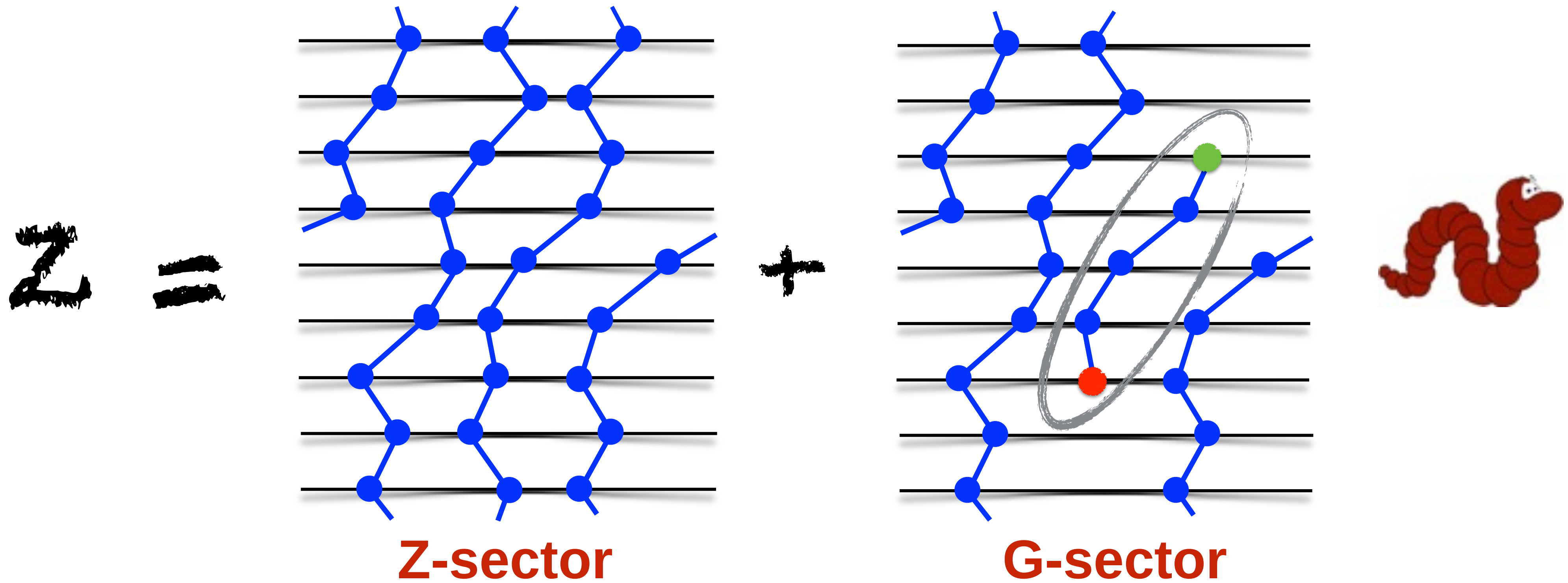
$$G(\vec{r}_1, \vec{r}_2, t) = \langle \mathcal{T}\{\hat{\psi}(\vec{r}_1, t)\hat{\psi}^\dagger(\vec{r}_2, 0)\}\rangle$$

$\hat{\psi}(\vec{r}, \tau)$ $\hat{\psi}^\dagger(\vec{r}, \tau)$ annihilation/creation operators

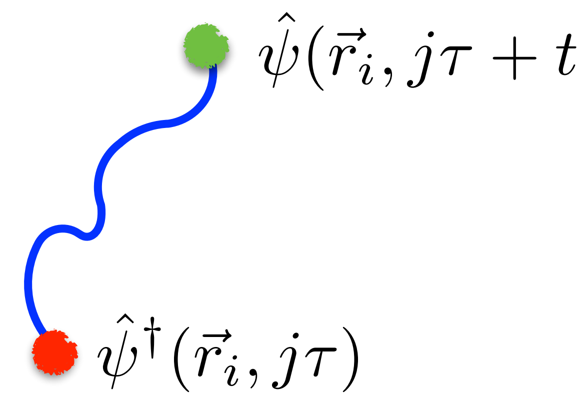


Z-sector
diagonal configurations

G-sector
off-diagonal configurations



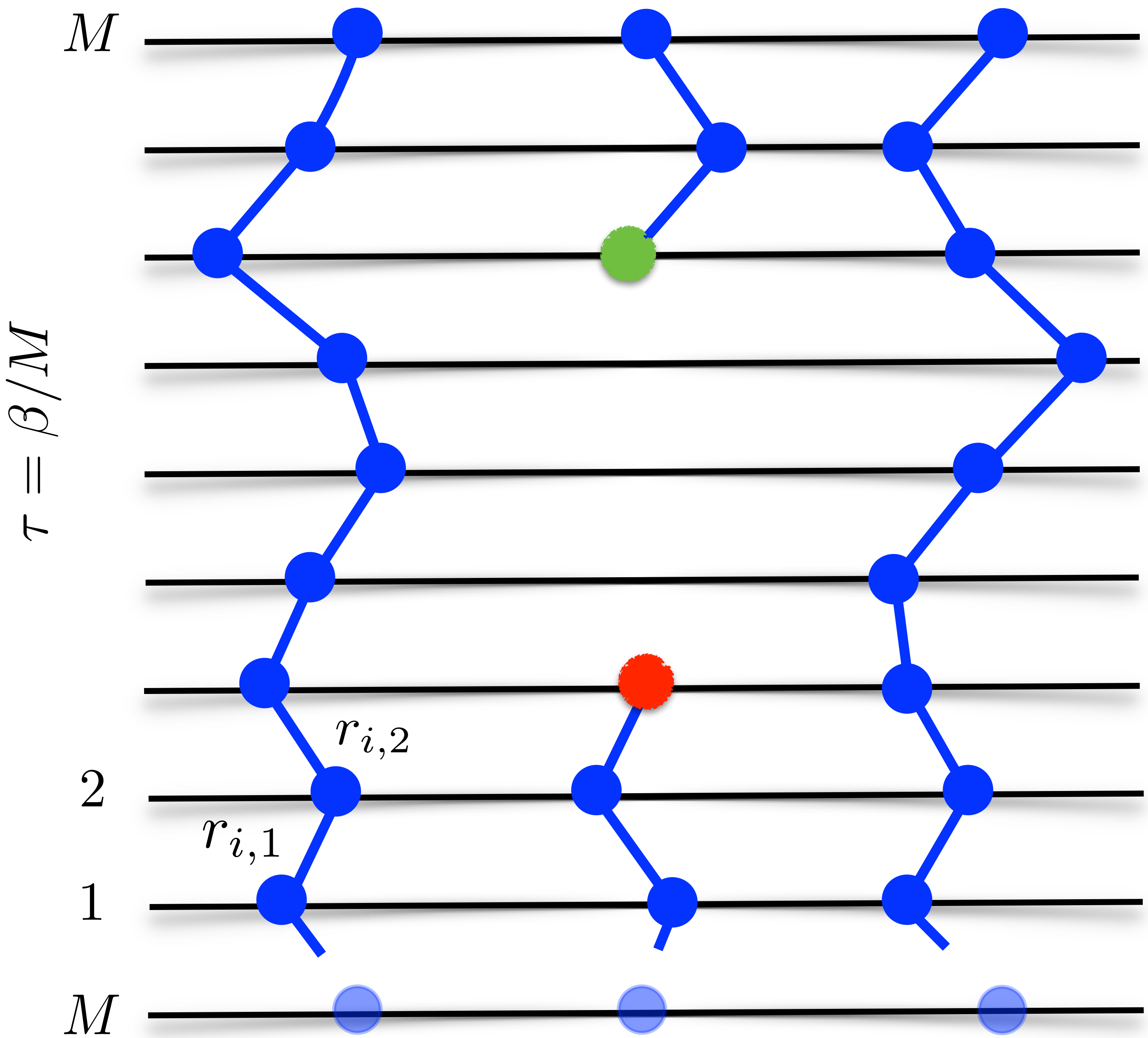
- Sampling the many-particle paths in Z- and G-sectors



- One opens path with two dangling ends (worm)
- in G-sector: a simple set of complementary moves, involving the worm

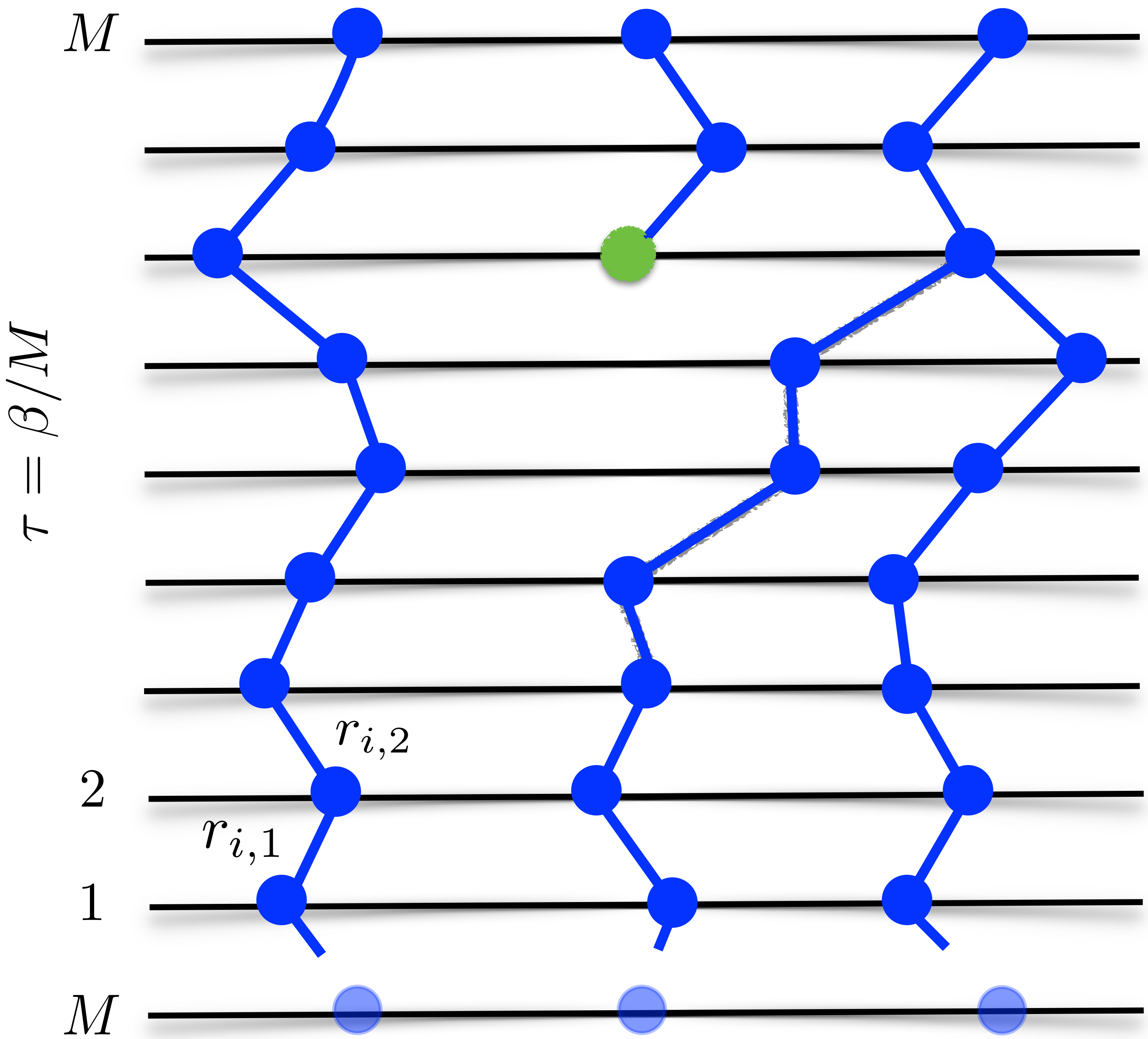
Boninsegni et al PRE 2006.

Sampling the G-sector: Swapping



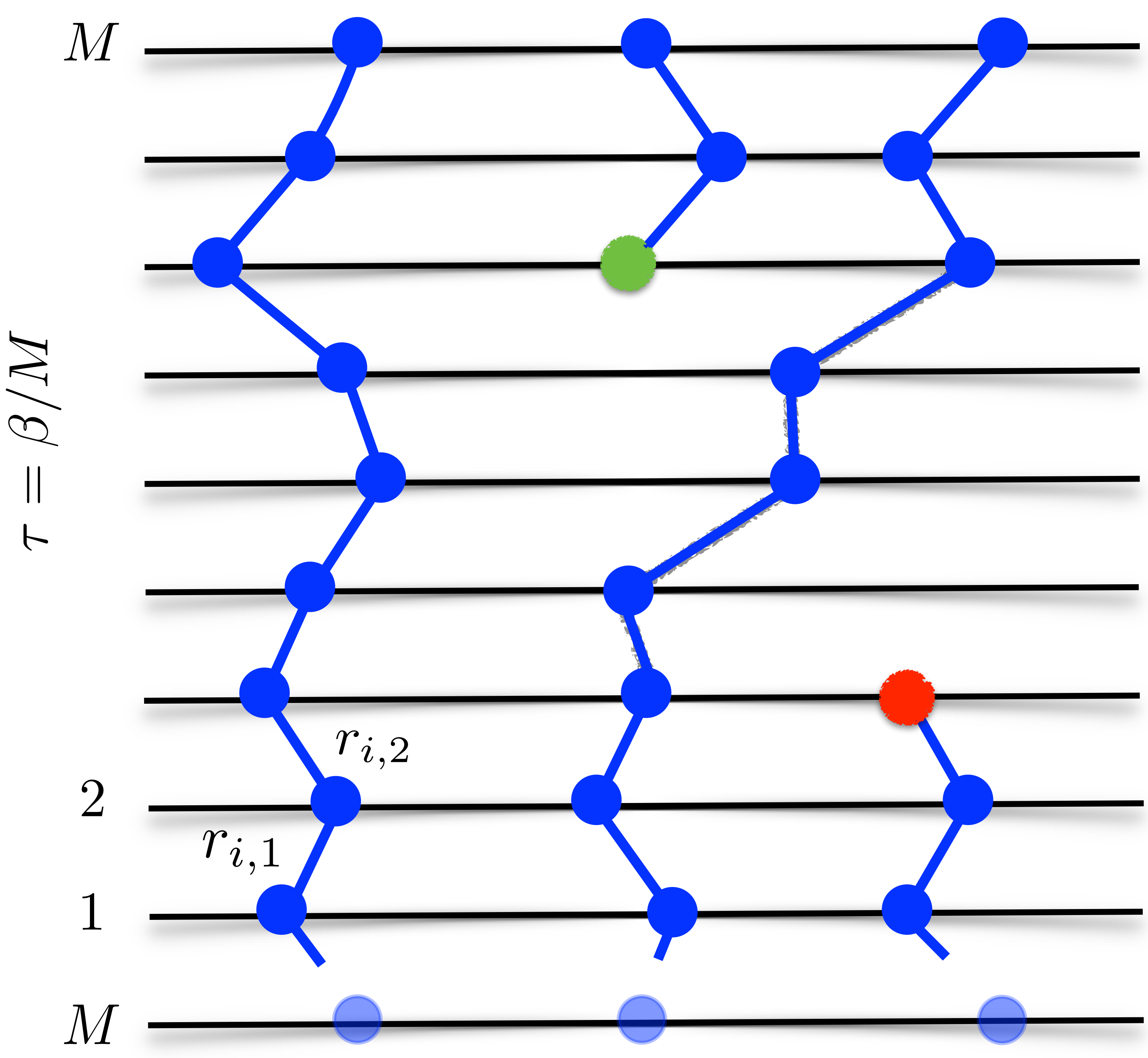
- Relatively high acceptance, also in hard core potentials
- May generate all possible many-body permutations
- Condensate's properties can be measured in the G-sectors

Sampling the G-sector: Swapping

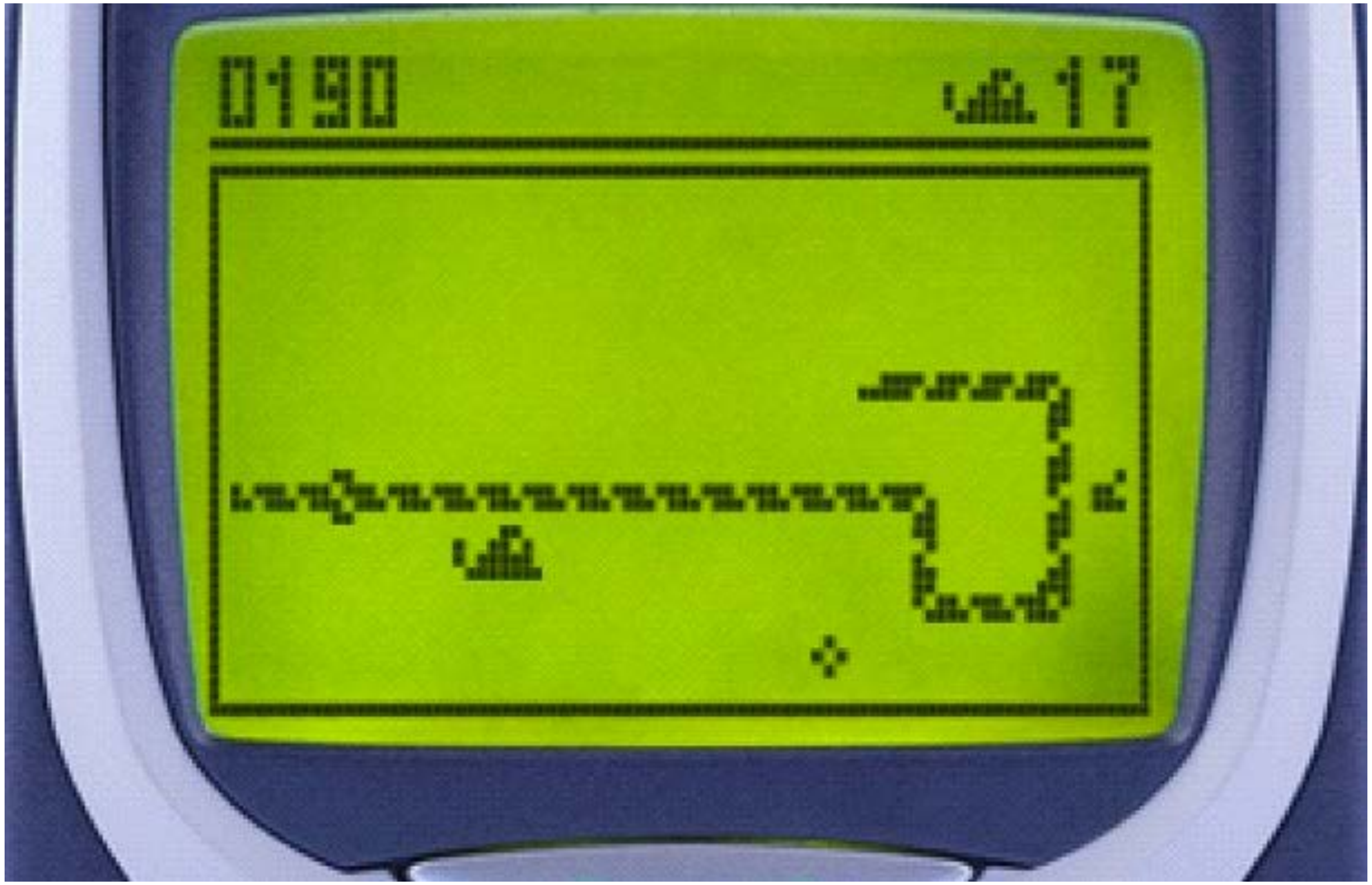


- Relatively high acceptance, also in hard core potentials
- May generate all possible many-body permutations
- Condensate's properties can be measured in the G-sectors

Sampling the G-sector: Swapping

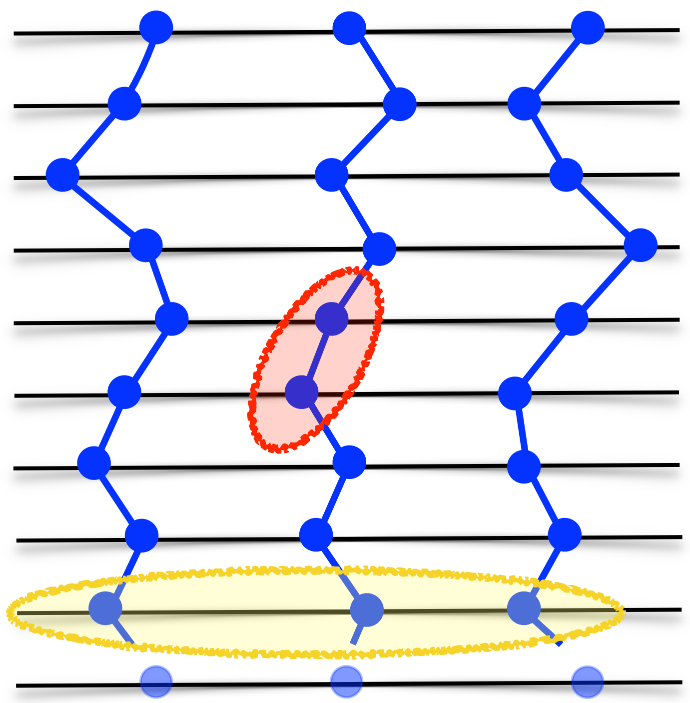


- Relatively high acceptance, also in hard core potentials
- May generate all possible many-body permutations
- Condensate's properties can be measured in the G-sectors



After a decent score (oops sampling) we can average some properties…

Z-sector



- Energy

$$\frac{E_{kin}}{N} \approx \frac{d}{2\tau} - \frac{1}{4\lambda\tau^2}\langle(\vec{r}_k - \vec{r}_{k+1})^2\rangle + \frac{\lambda\tau^2}{9}\langle(\nabla V(R_{2k}))^2\rangle$$

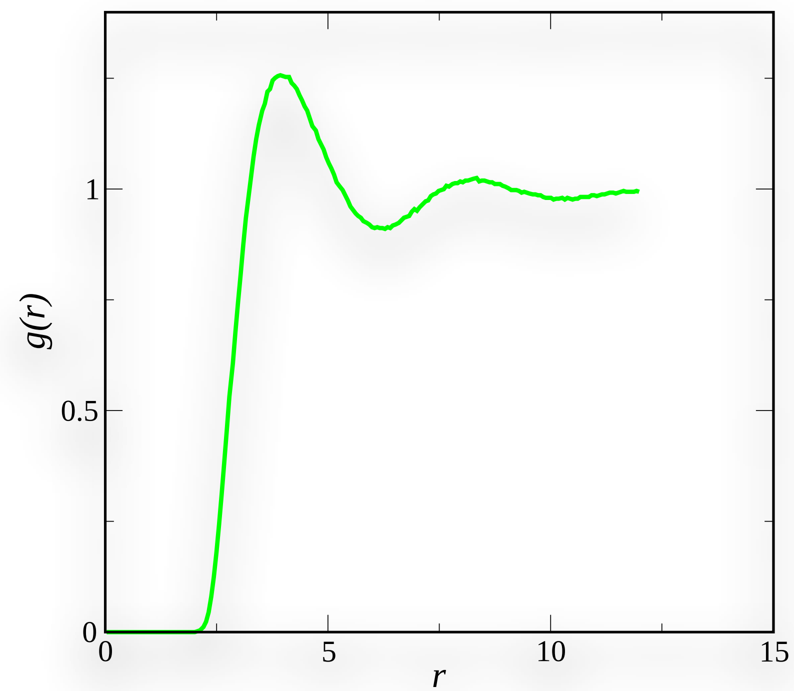
$$\frac{E_{pot}}{N} \approx \langle V(R_{2k-1})\rangle$$

- Pressure as well

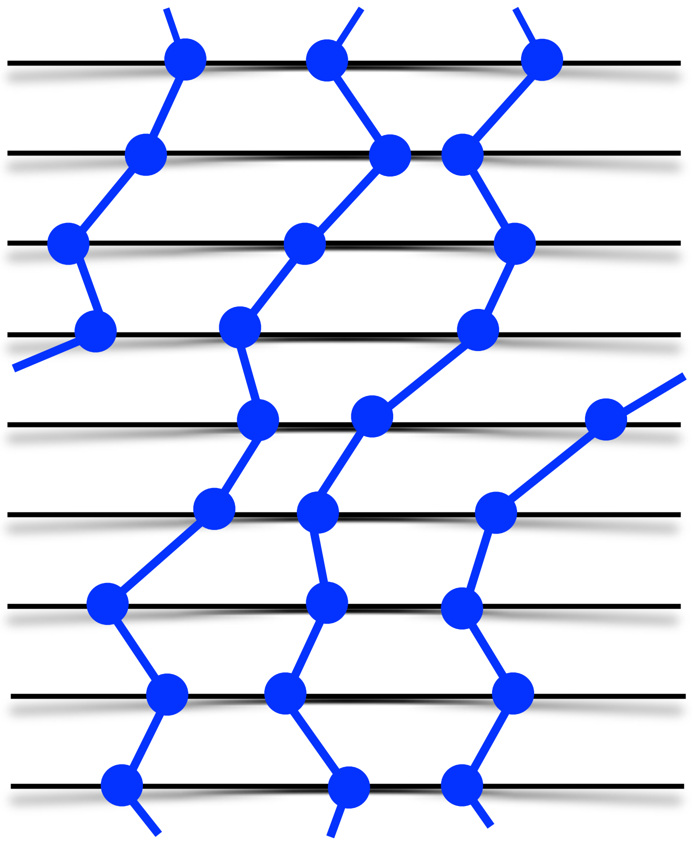
$$\mathcal{P}(N,V,\beta) = \frac{1}{\beta Z}\frac{\partial Z(N,V,\beta)}{\partial V}$$

- Pair correlation function

$$g_2(\vec{r}) = \frac{V^2}{N}\langle\sum_{i\neq j}\delta\left(\vec{r} - \vec{r}_{ij}\right)\rangle$$



example with a radial symmetry



Superfluidity

- Different response of normal/super fluid component to the boundary of the container
- Under rotation the superfluid component remains at rest.
- Non-classical rotational Inertia $f_s = 1 - \frac{I_q}{I_c}$
- We can evaluate f_s using topological properties of the system such as the *winding number*

different from zero if some permutation cycle winds around the periodic boundary condition

$$\mathbf{W} = \sum_i (\vec{r}_{\mathcal{P}i} - \vec{r}_i)$$

averaged over a simulation in *d*-dimension:

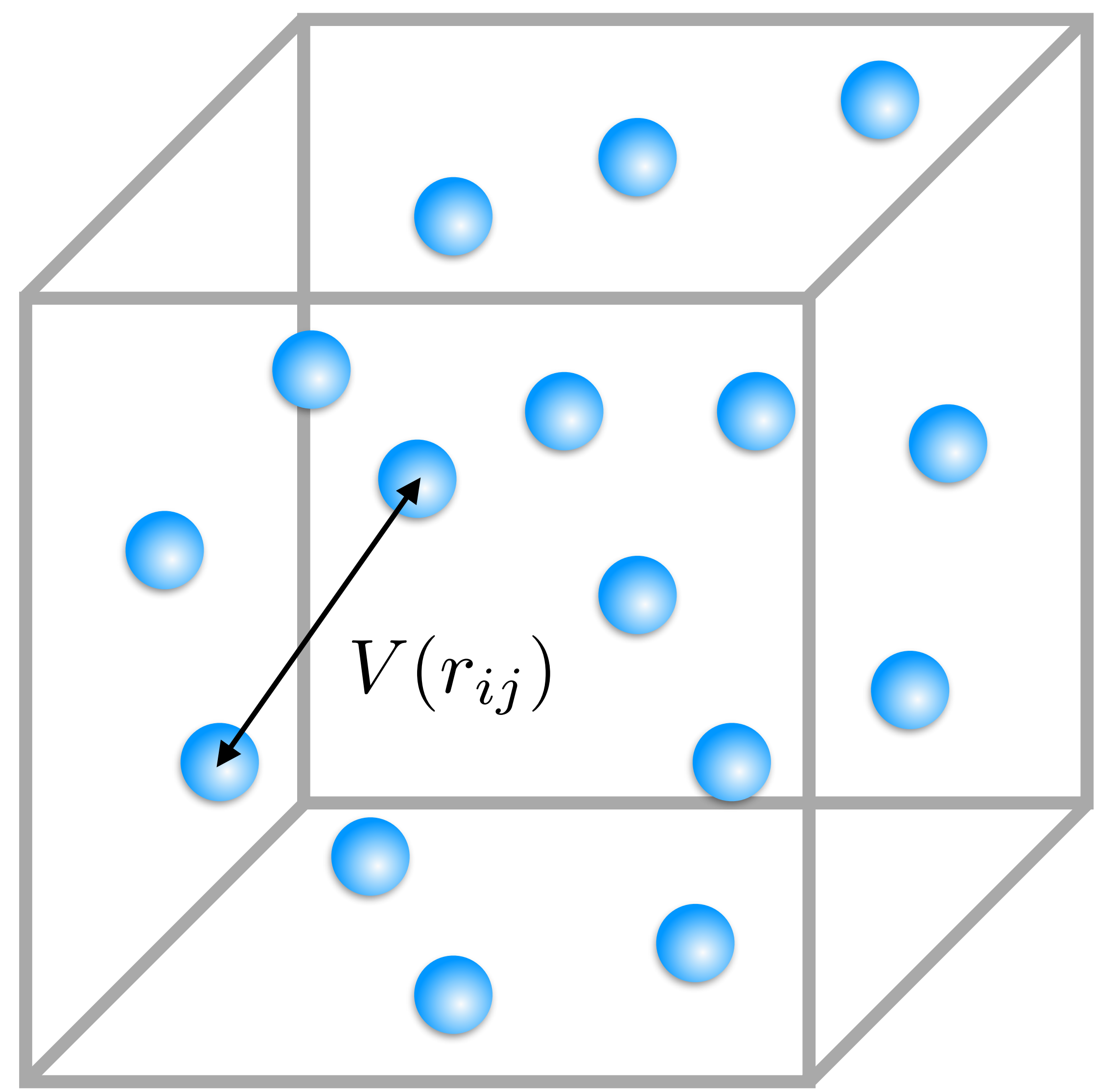
$$f_s = \frac{1}{2\lambda} \frac{\langle \mathbf{W}^2 \rangle L^{2-d} d}{\rho d \beta}$$

Just recapping the system Hamiltonian

$$\mathcal{H} = -\frac{\hbar^2}{2m}\sum_{i}^{N}\nabla_i^2 + \sum_{i<j}^{N} V(\vec{r}_i - \vec{r}_j)$$

- N-bosons
- PBC applied
- N=100÷400
- T≪Tc (ground state limit)
- Gas parameter ≈10^{-5} ÷10^{-1}

$$V(r_{ij}) = \begin{cases} \frac{d^2}{r_{ij}^3}\left(1 - 3\cos^2\vartheta\right), & \text{if } r_{ij} \geq a, \\ \infty, & r_{ij} < a. \end{cases}$$

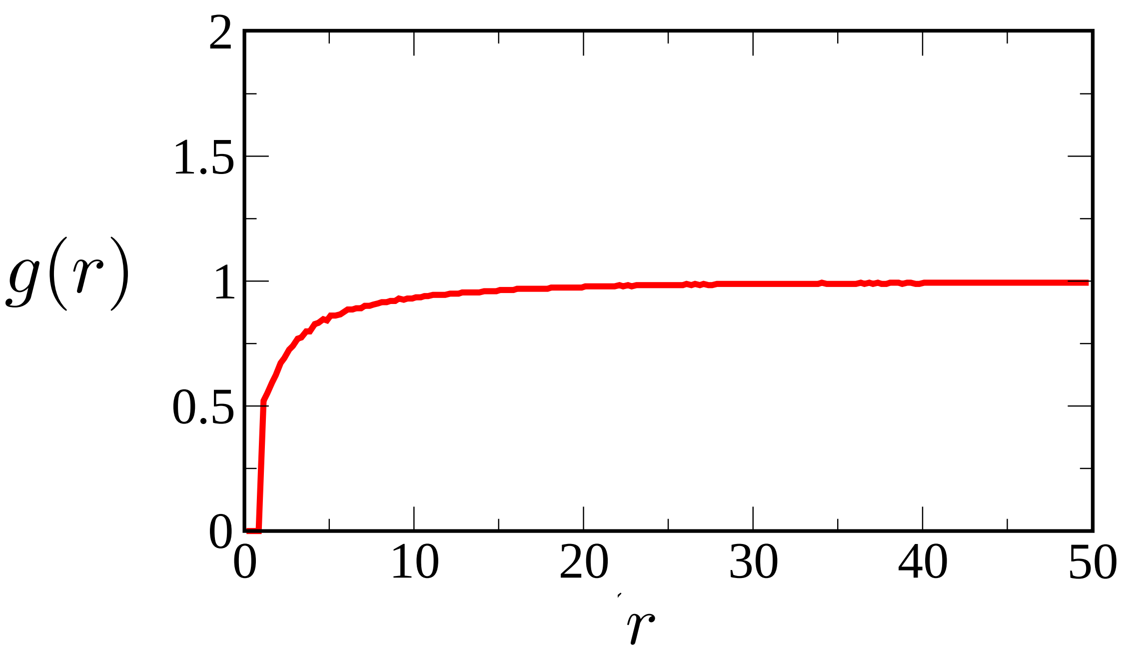


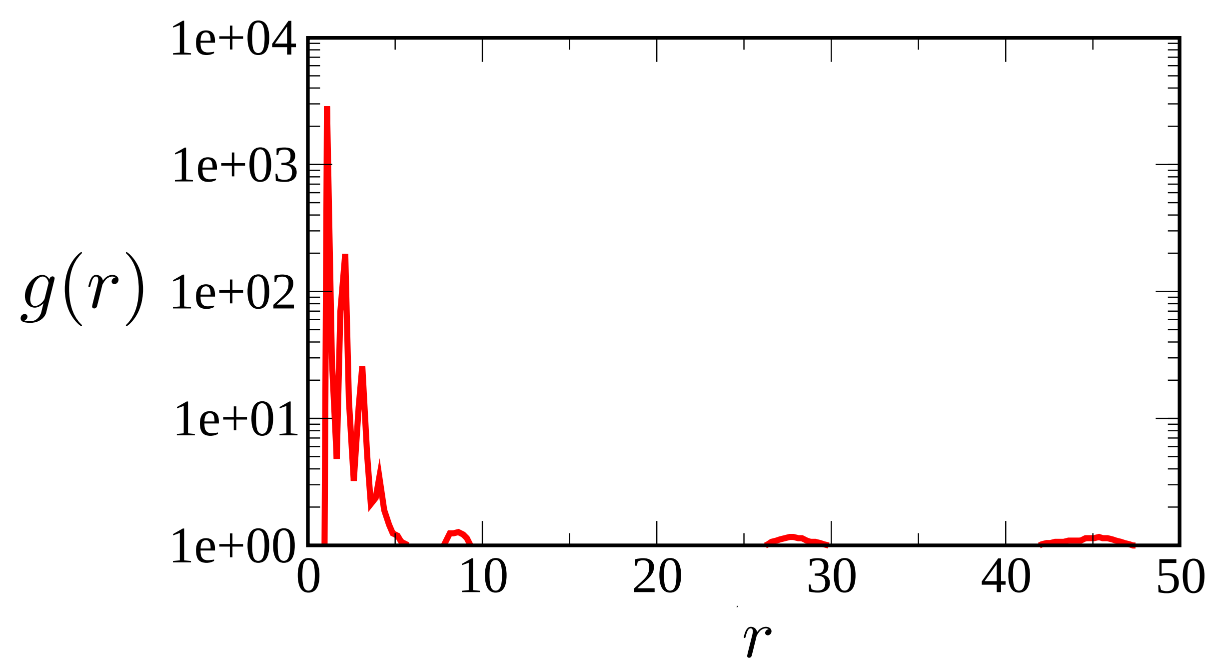
FC, Cappellaro, Salasnich, Macrì, submitted

Structural properties by increasing the dipolar length

$$g(r) \propto \langle \sum_i \sum_{j \neq i} \delta\left(r - r_{ij}(t)\right) \rangle$$

$$na^3 \approx 10^{-4}$$





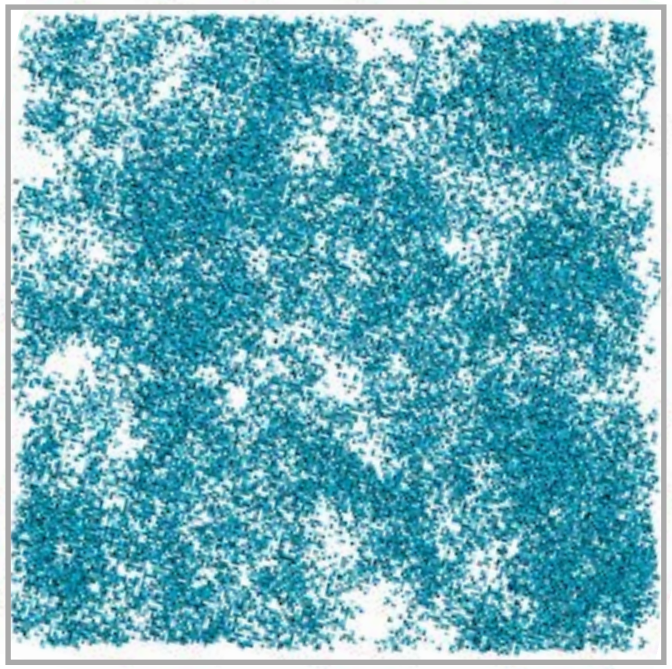
$\varepsilon \approx 1$

$a(d)$

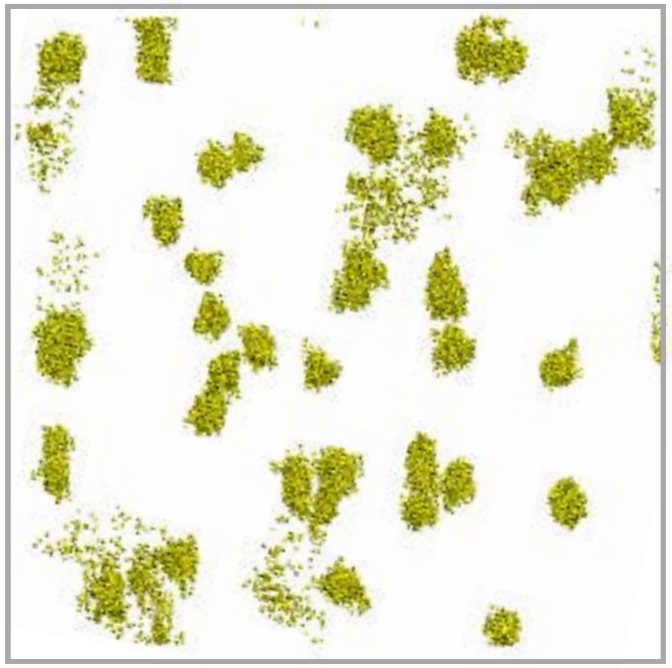
Structural properties by increasing the dipolar length

$$g(r) \propto \langle \sum_i \sum_{j \neq i} \delta\left(r - r_{ij}(t)\right) \rangle$$

$na^3 \approx 10^{-4}$



homogeneous
superflid



unstable
phase

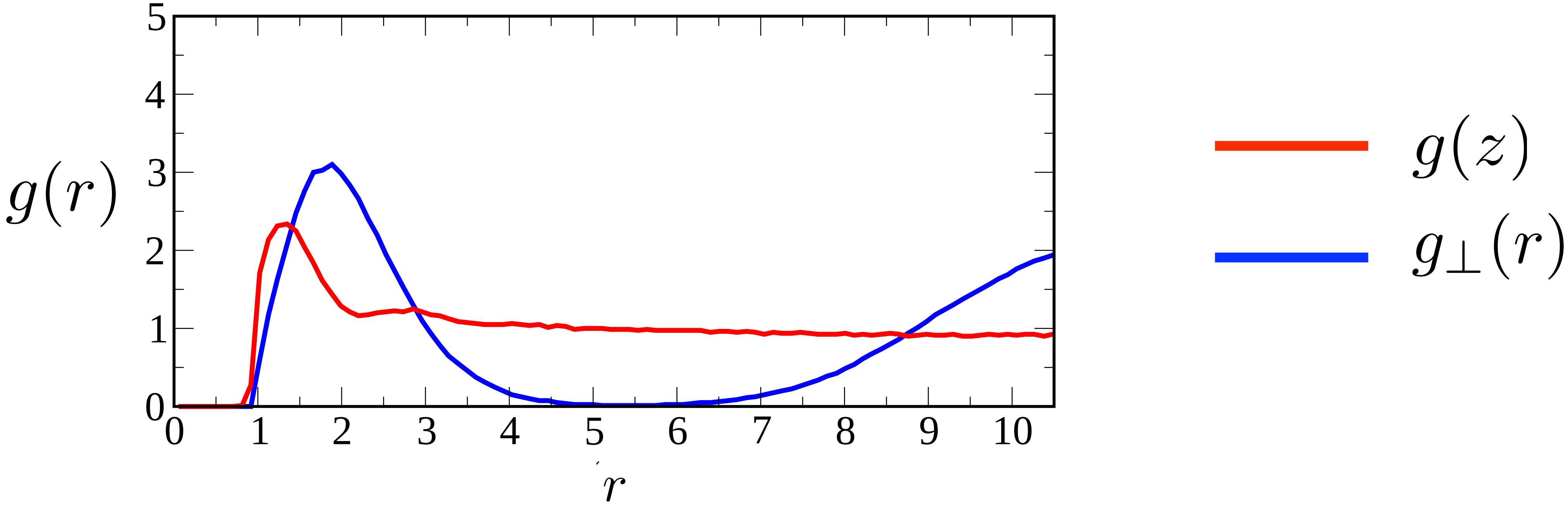
$\varepsilon \approx 1$

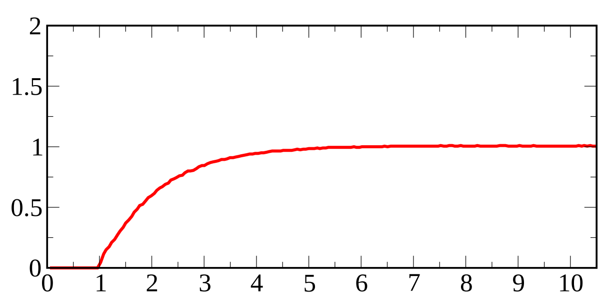
$a(d)$

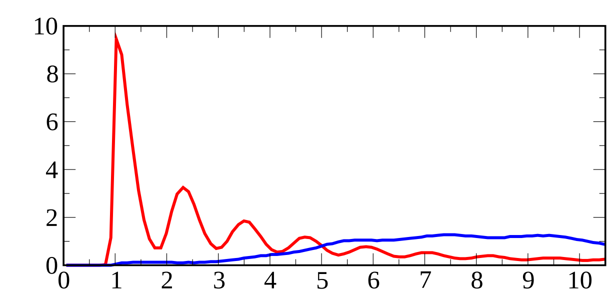
Structural properties by increasing the dipolar length

$$na^3 \approx 10^{-2}$$

$$g(r) \propto \langle \sum_i \sum_{j \neq i} \delta\left(r - r_{ij}(t)\right) \rangle$$





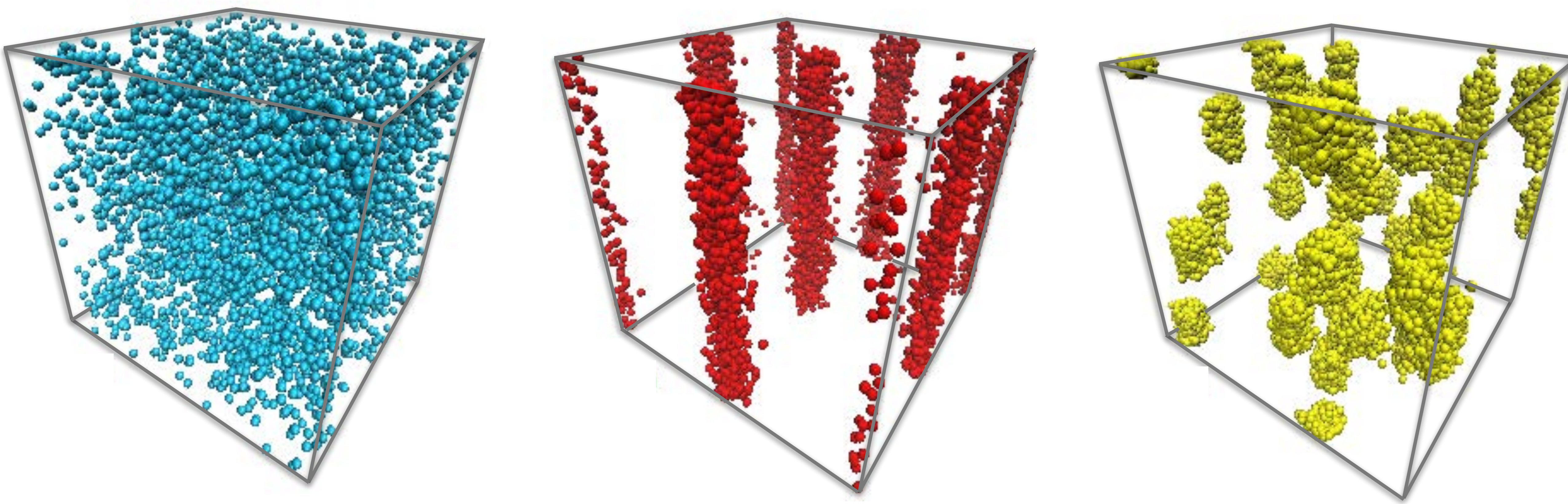


$\varepsilon \approx 1$ $\varepsilon \approx \varepsilon^\star$ $a(d)$

Structural properties by increasing the dipolar length

$$na^3 \approx 10^{-2}$$

$$g(r) \propto \langle \sum_{i} \sum_{j \neq i} \delta \left(r - r_{ij}(t) \right) \rangle$$



homogeneous superflid

filaments

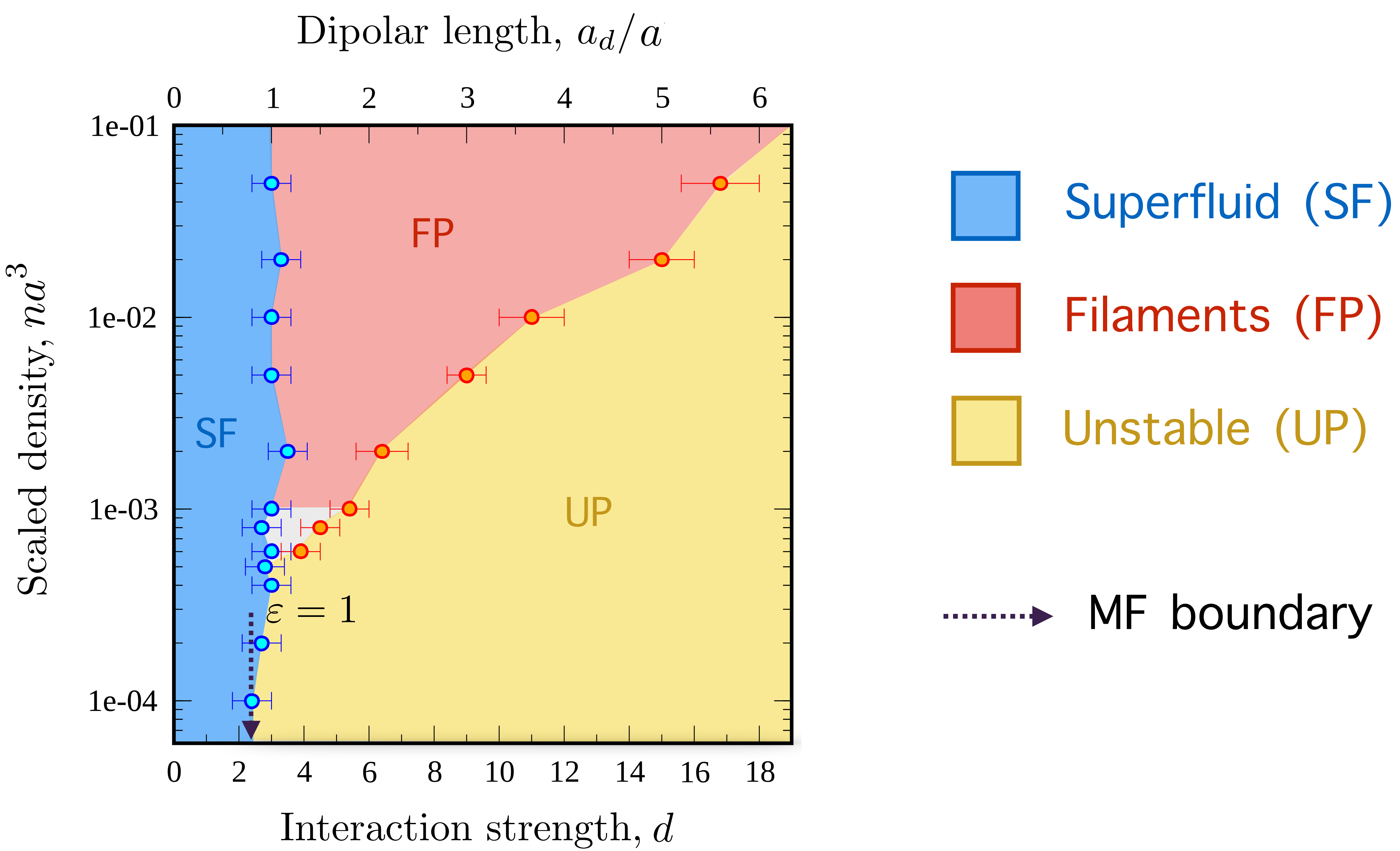
unstable phase

$\varepsilon \approx 1$

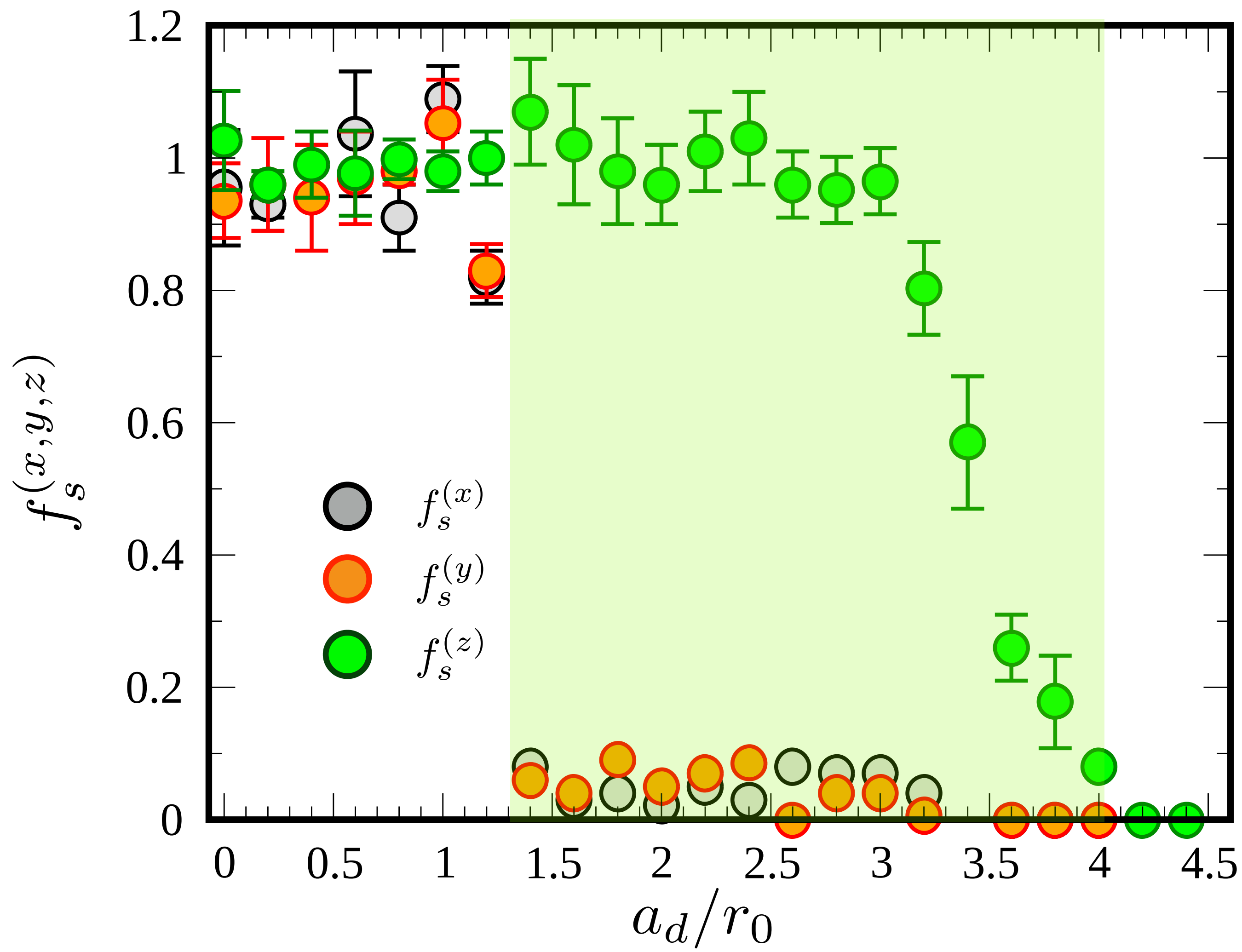
$\varepsilon \approx \varepsilon^{\star}$

$a(d)$

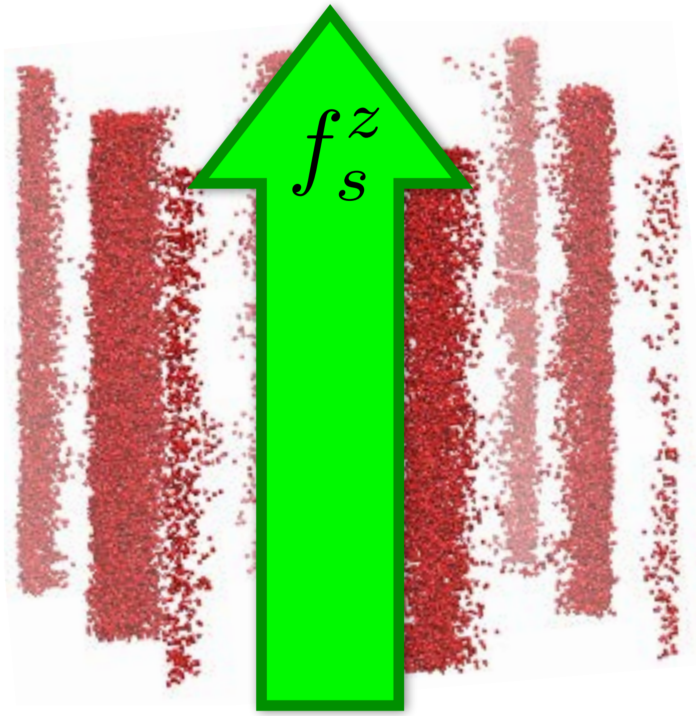
Phase diagram of dipolar bosons in 3d



Superfluidity as a function of dipolar length



$na^3 \approx 10^{-2}$

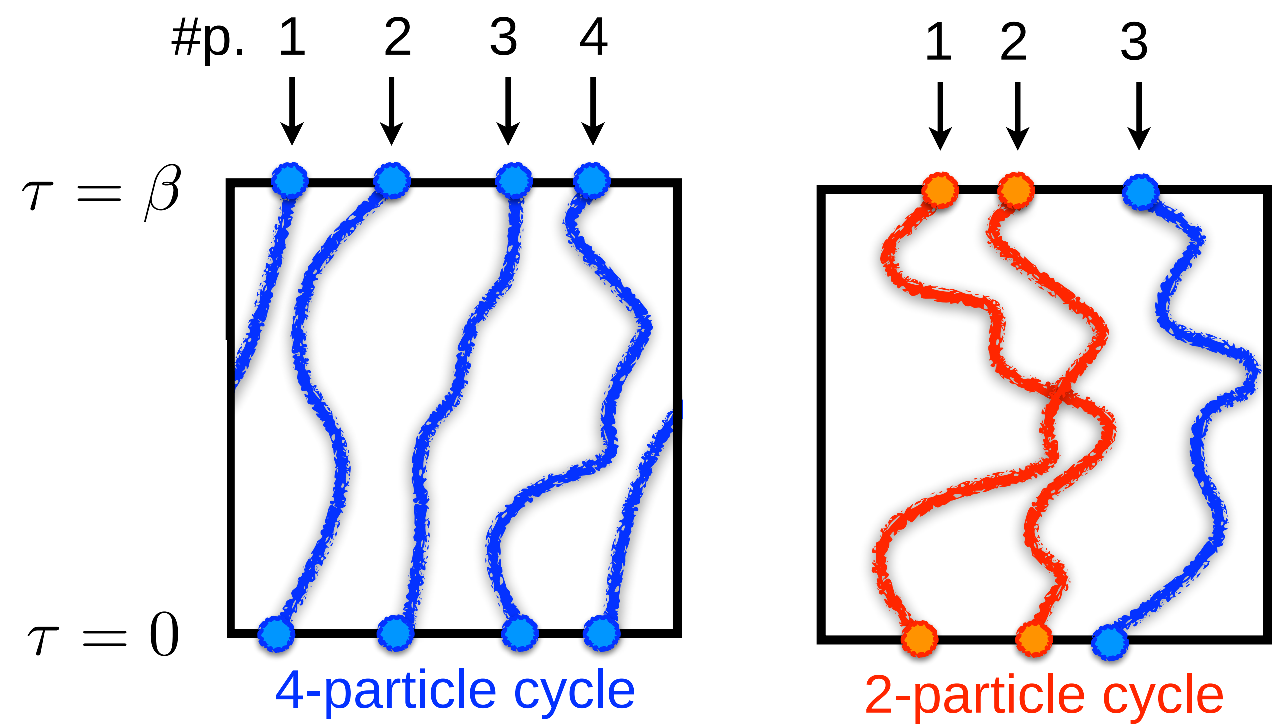


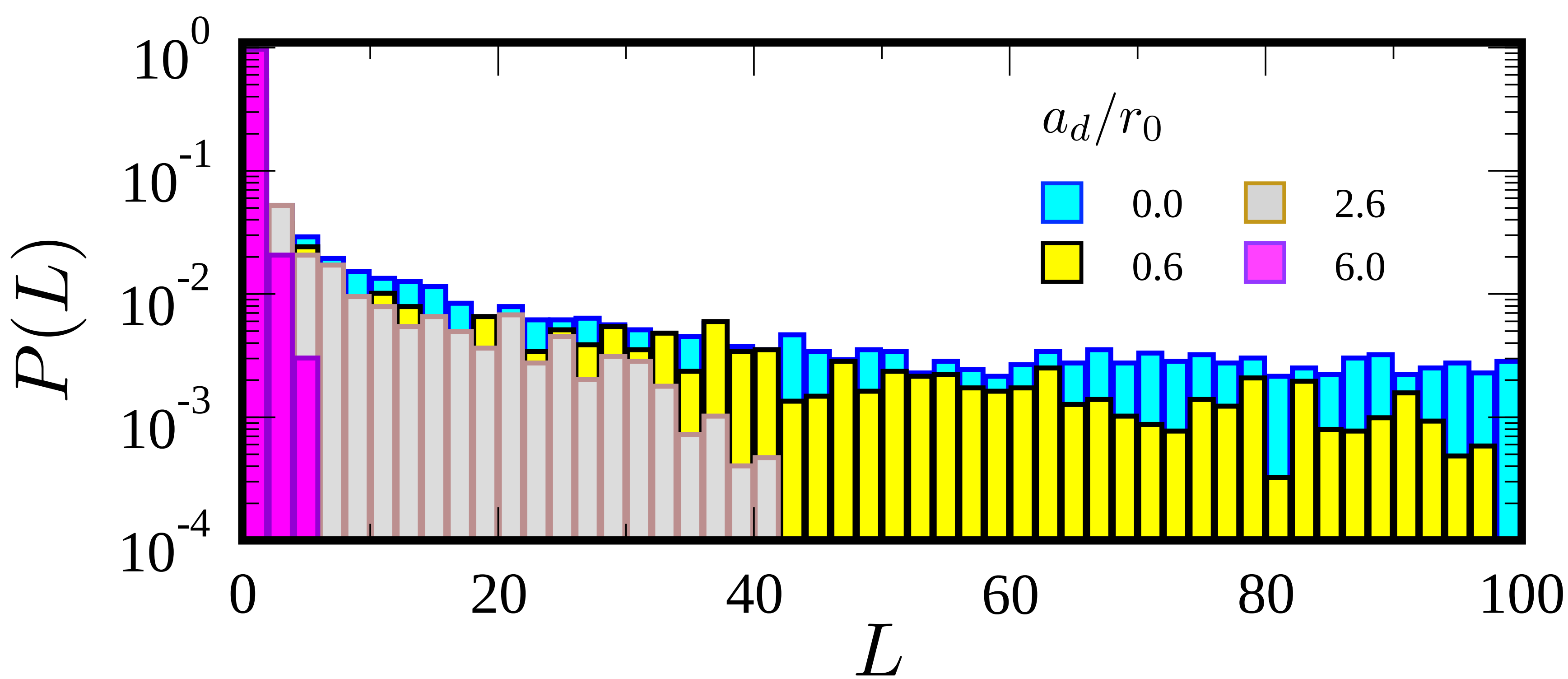
- Uniform SF: $f_s=1$
- Crossing SF-FI phase f_s is anisotropic
- $f_s^{(x)} = f_s^{(y)} \approx 0$ and $f_s^{(z)} = 1$
- **Each filaments is phase coherent**
- Global coherence is not observed

Permutation cycles the whole 3d space

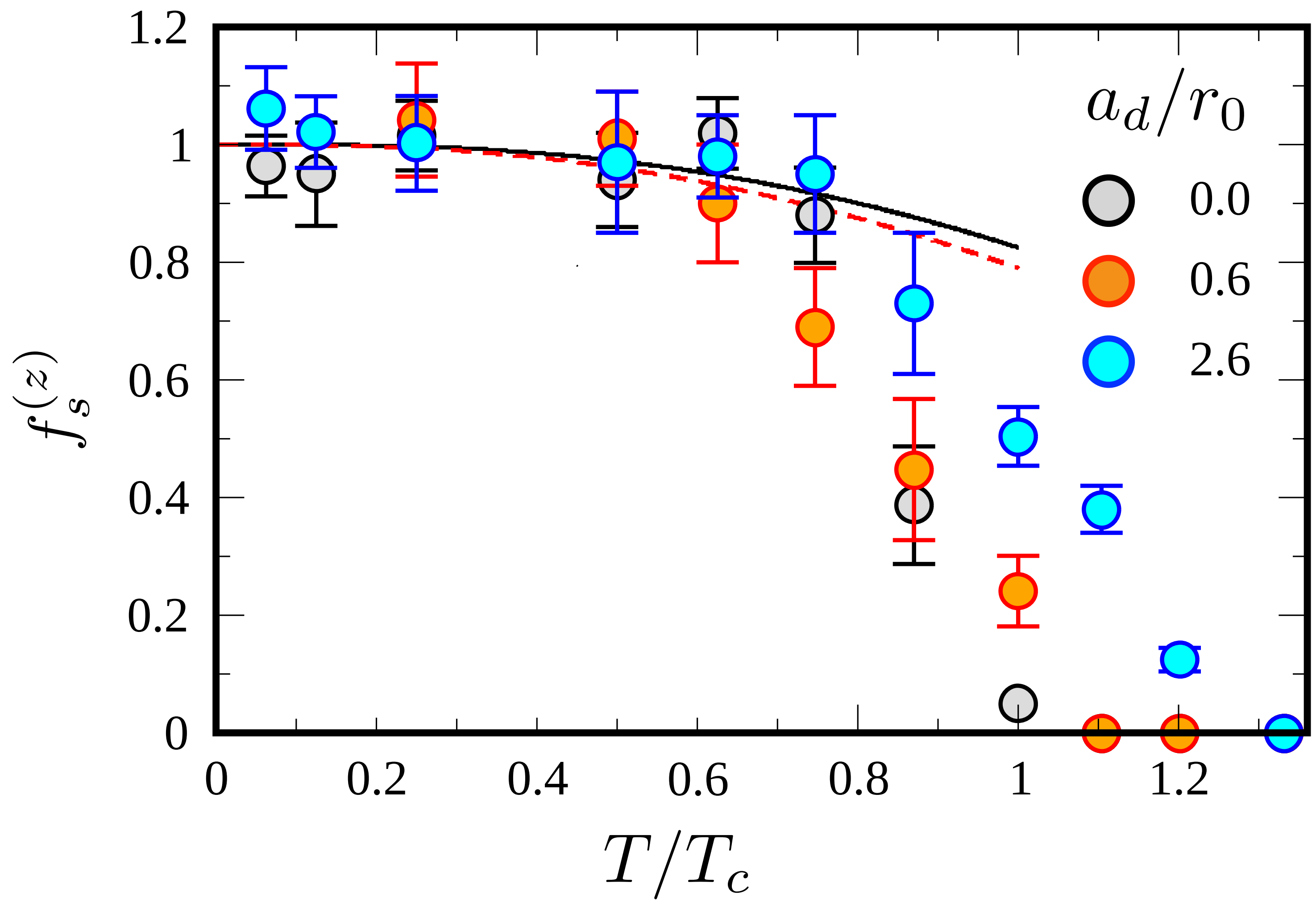
World lines are β-periodic that is $\mathbf{R}_\beta$=$\mathbf{R}_0$ but individual particle positions can undergo exchanges

$$\rho^{\mathrm{Bose}}\left(\mathbf{R}_i, \mathbf{R}_{i+1}, \beta\right) = \frac{1}{N!}\sum_{\mathcal{P}} \rho\left(\mathbf{R}_i, \mathcal{P}\left(\mathbf{R}_{i+1}\right), \beta\right)$$





Superfluidity along *z* at finite temperature

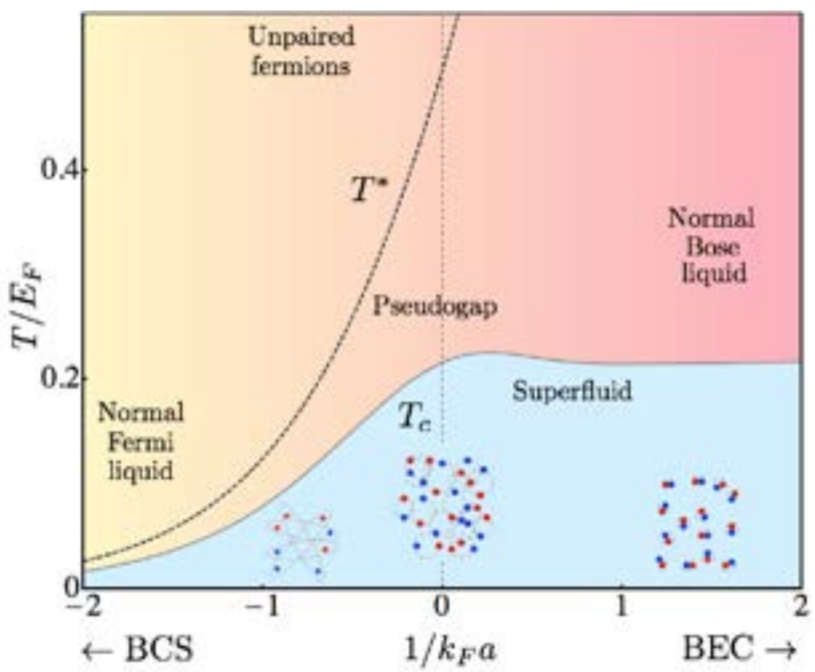


$$k_B T_c = \frac{2\pi}{\zeta\left(3/2\right)^{2/3}}\left(na^3\right)^{2/3}$$

For a non-interacting Bose gas

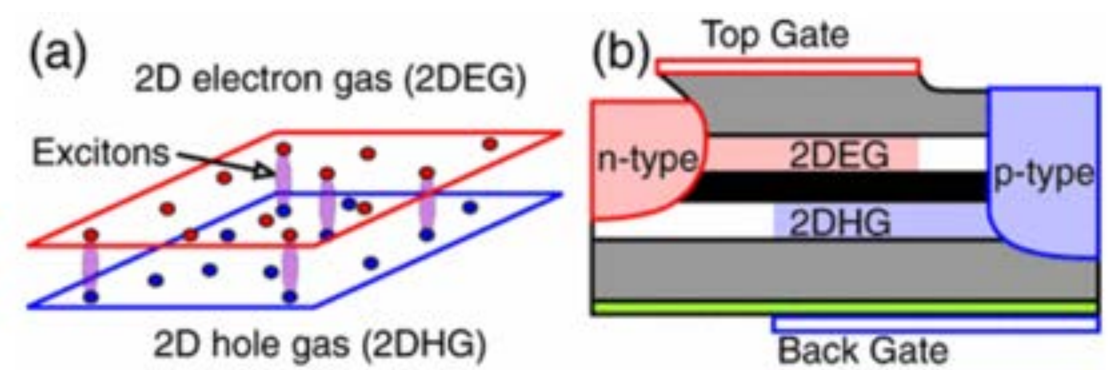
- $a_d \to 0$ agreement with previous QMC, Pilati 2008
- $a_d > 0$ stability at finite T for SF and FI
- Tc as a function of a_d: to be completed

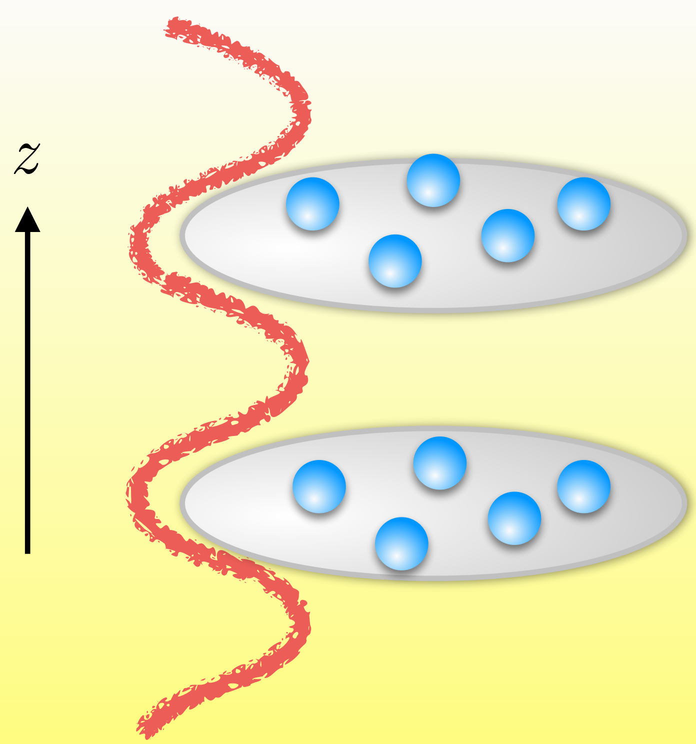
Dipolar bosons on bi-layers



- Fermions on bi-layers: interlayer pairing, BCS to BEC transition.

- Electron-hole bilayers in semiconductors coupled quantum wells. Seasons 2009.

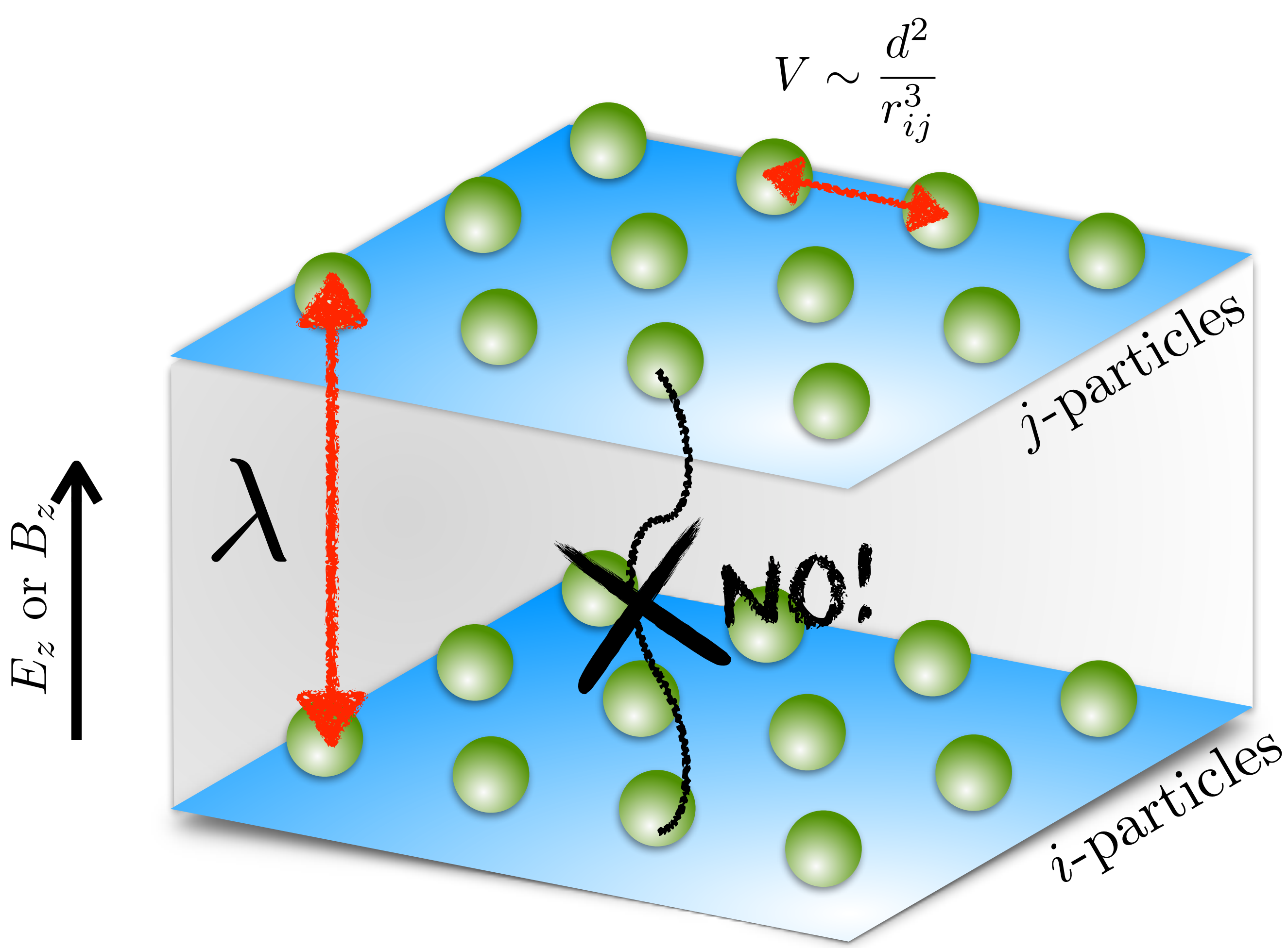




- Dipolar Bosons: adding up an in-plane SF phase. Ex: stack of pancake-shaped dipolar-condensates with one-dimensional optical lattices.

Model

$$\mathcal{H} = \frac{\hbar^2}{2m}\sum_{i}^{N/2}\nabla_i^2 + \frac{\hbar^2}{2m}\sum_{j}^{N/2}\nabla_j^2 + \sum_{i<i'}\frac{d^2}{r_{ii'}^3} + \sum_{j<j'}\frac{d^2}{r_{jj'}^3} + \sum_{i<j}\frac{d^2(r_{ij}^2 - 2\lambda^2)}{(r_{ij}^2+\lambda^2)^{5/2}}$$



$$\varepsilon = \frac{\hbar^2}{ma^2} \qquad \bar{a} = \frac{md^2}{\hbar^2} \qquad r_s = \left(n\bar{a}^2\right)^{-1/2}$$

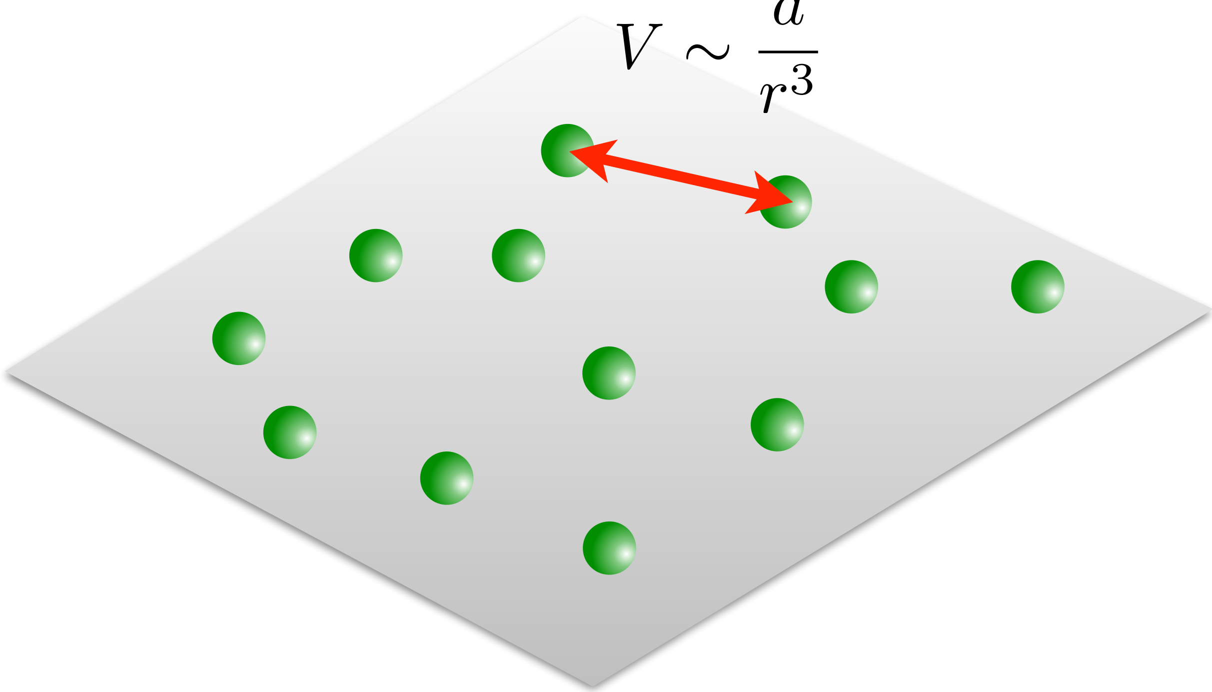
- r_{ij}-projection ***i-j*** distance
- N-Bosons, m mass, d dipole moment (**E** or **B**)
- Characteristic energy and length: ε and $\bar{a}$
- Mean inter particle distance and layer distance: r_s and λ
- No hopping between layers

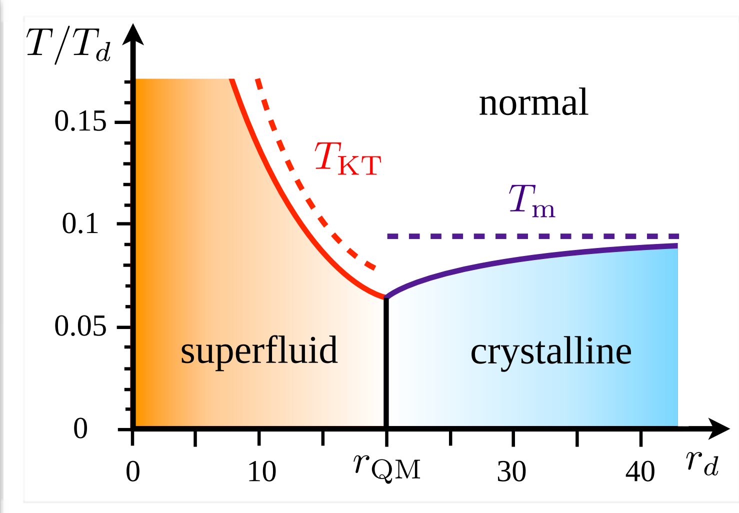
FC, Wang Boninsegni, submitted

Dipolar bosons in two-dimensions

Off Lattice

$$V \sim \frac{d}{r^3}$$



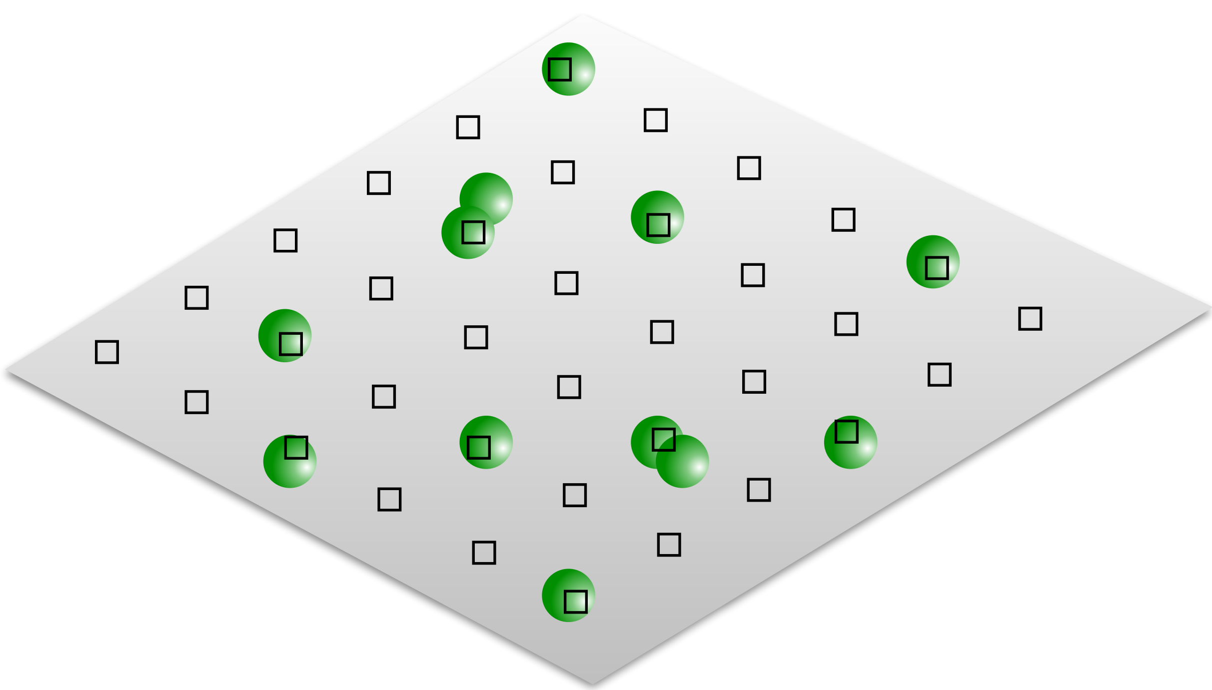


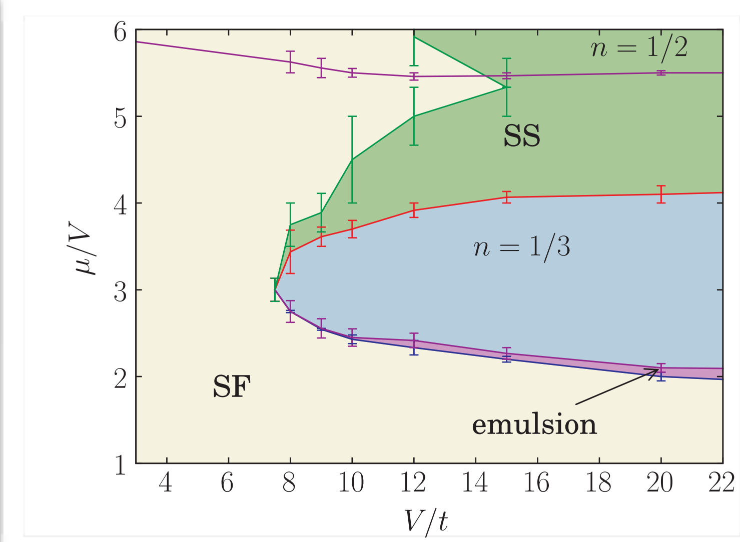
Moroni, Boninsegni PRL 2014

Büchler et al PRL 2007

Astrakharchik et al PRL 2007

On Lattice





Pollet et al PRL 2010

Danshita PRA 2010

Boninsegni PRL 2005

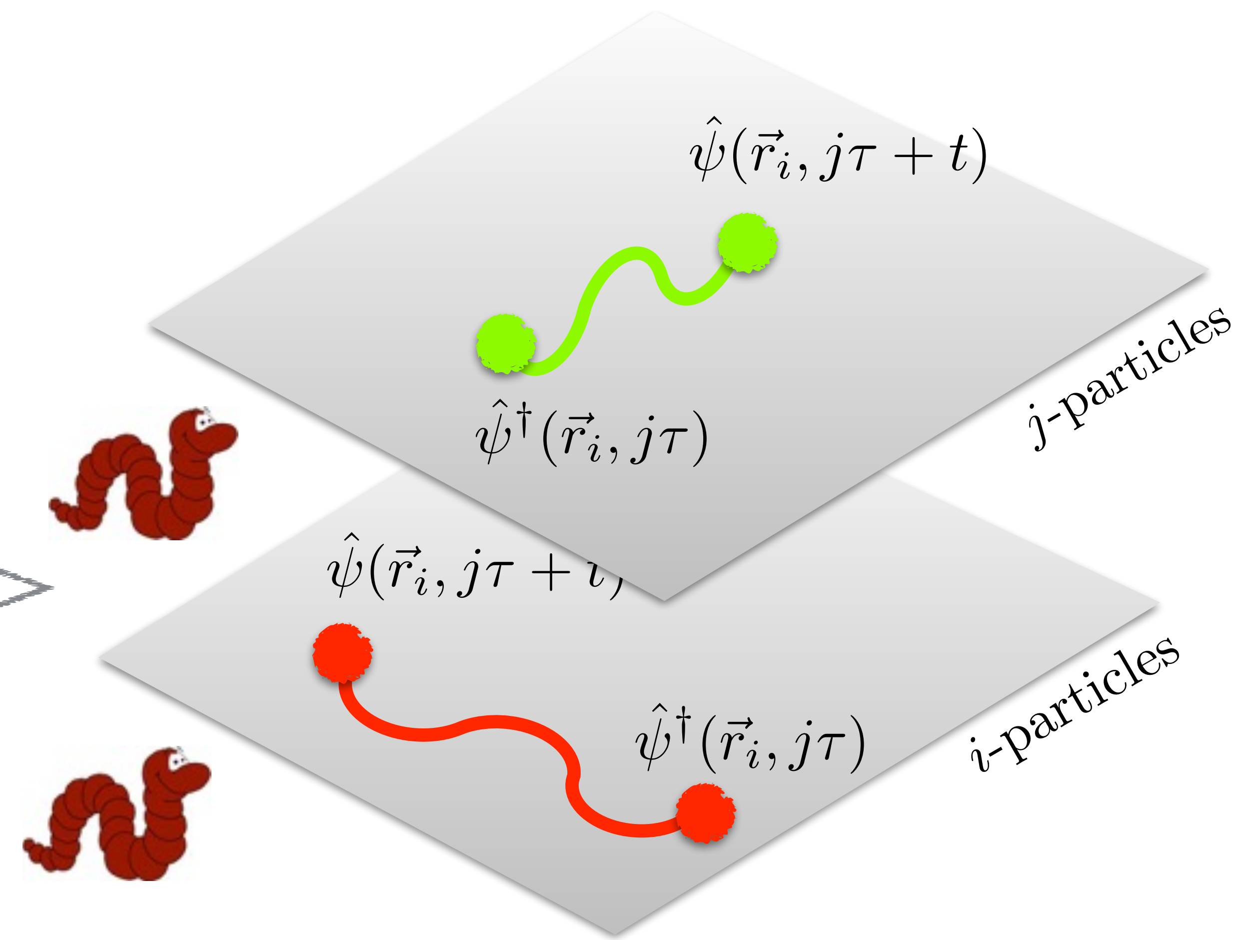
Again on QMC

- Double worm sampling (pairing)
- In-plane superfluidity:

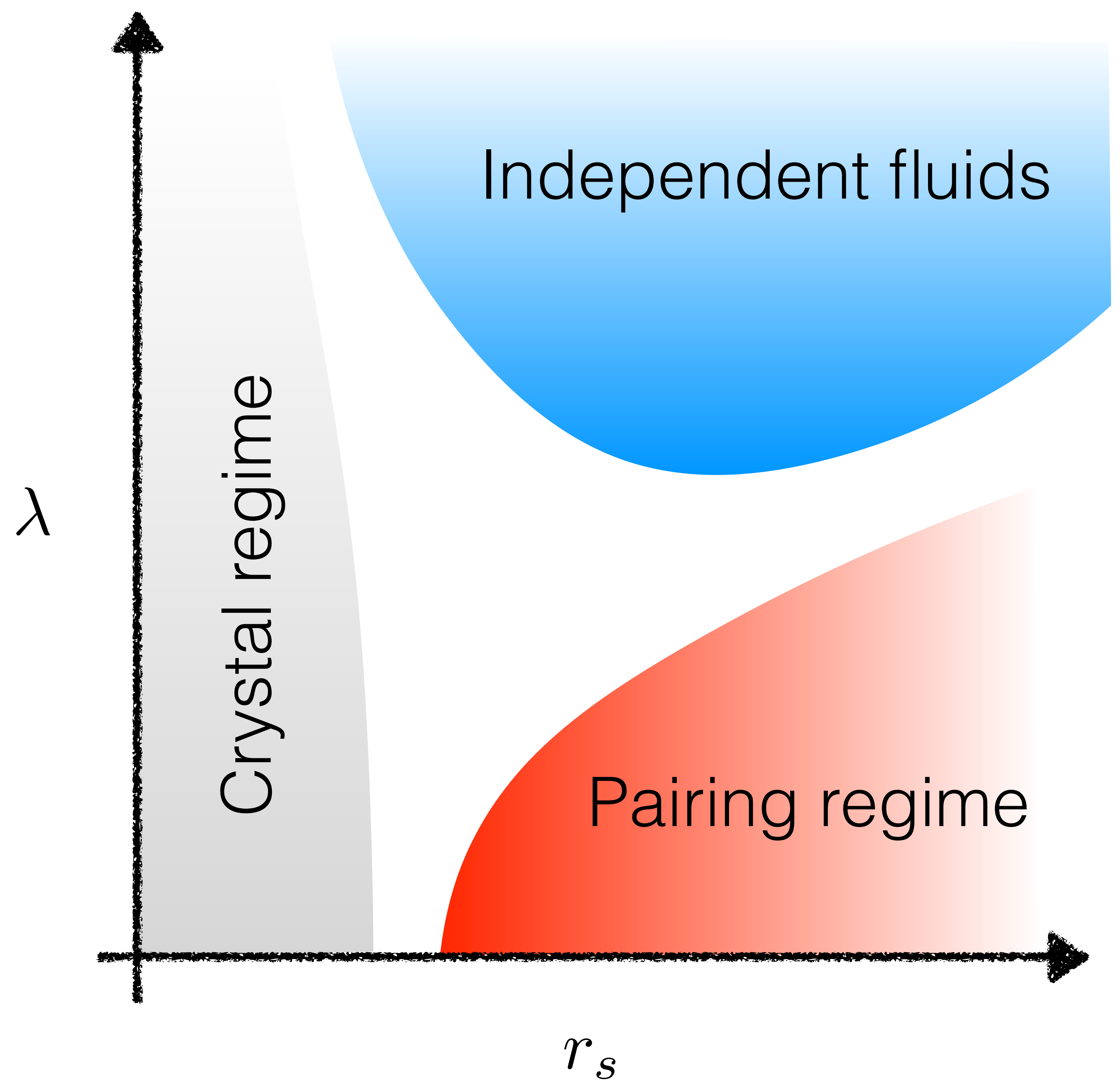
$T \to 0$

$f_s = 1.0$ decoupled superfluids

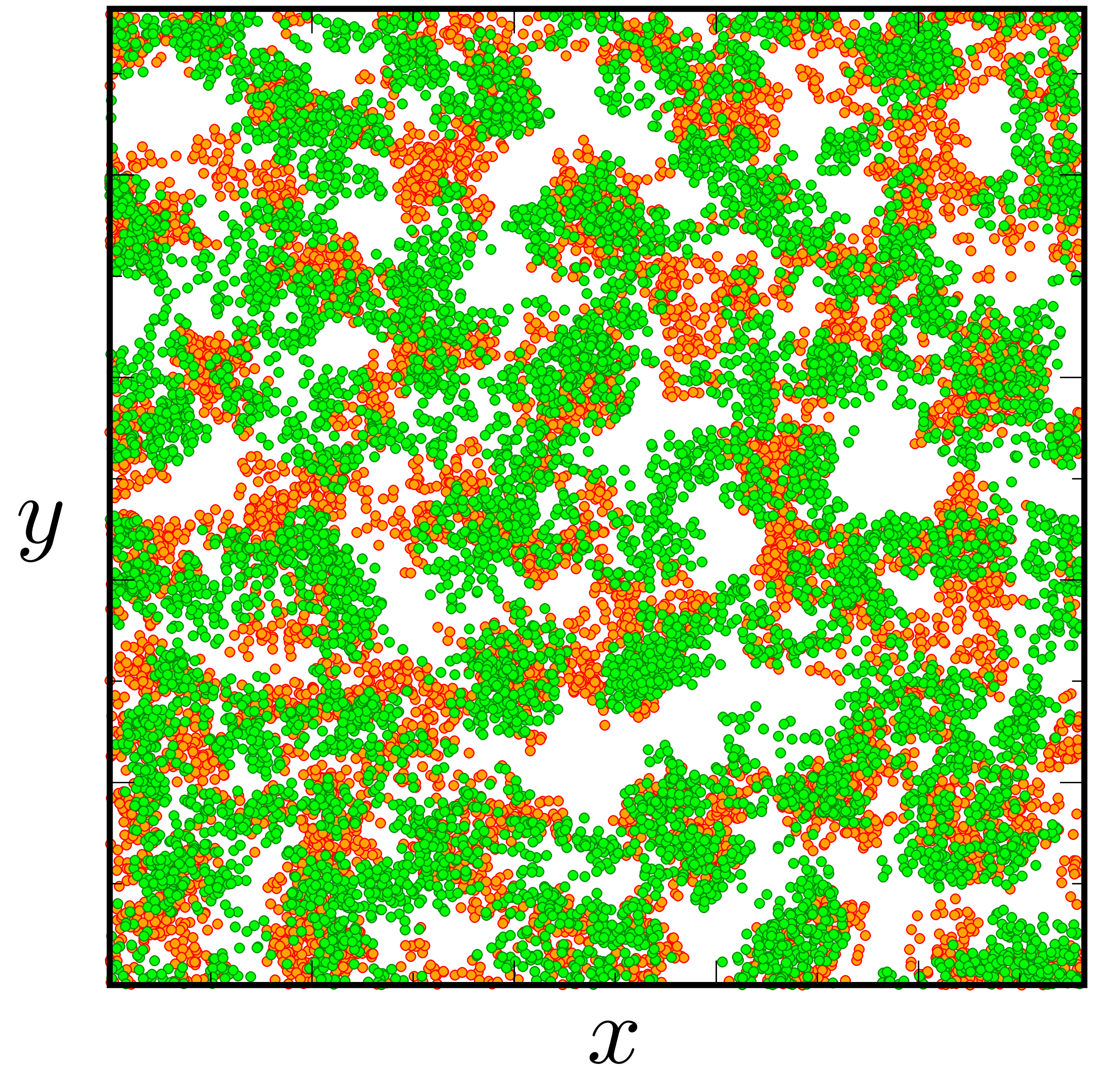
$f_s = 0.5$ superfluid dimers (M=2m)

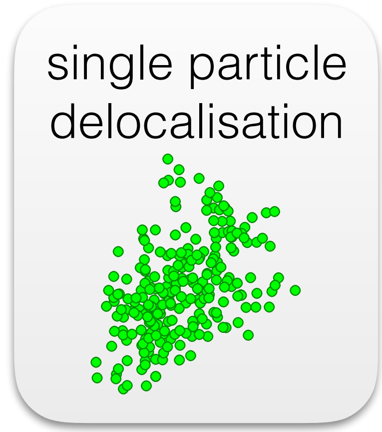


…what might we expect from a dipolar system with such a geometry?



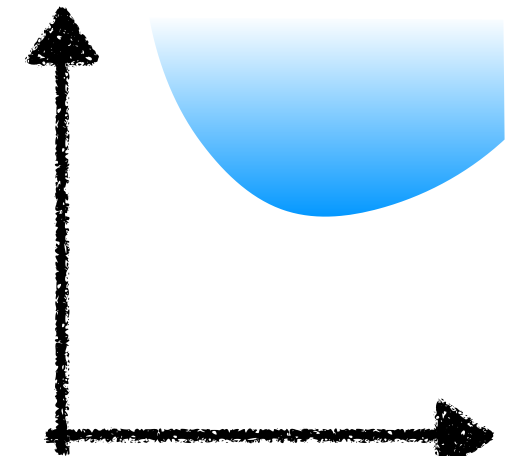
Snapshot configurations (world lines)



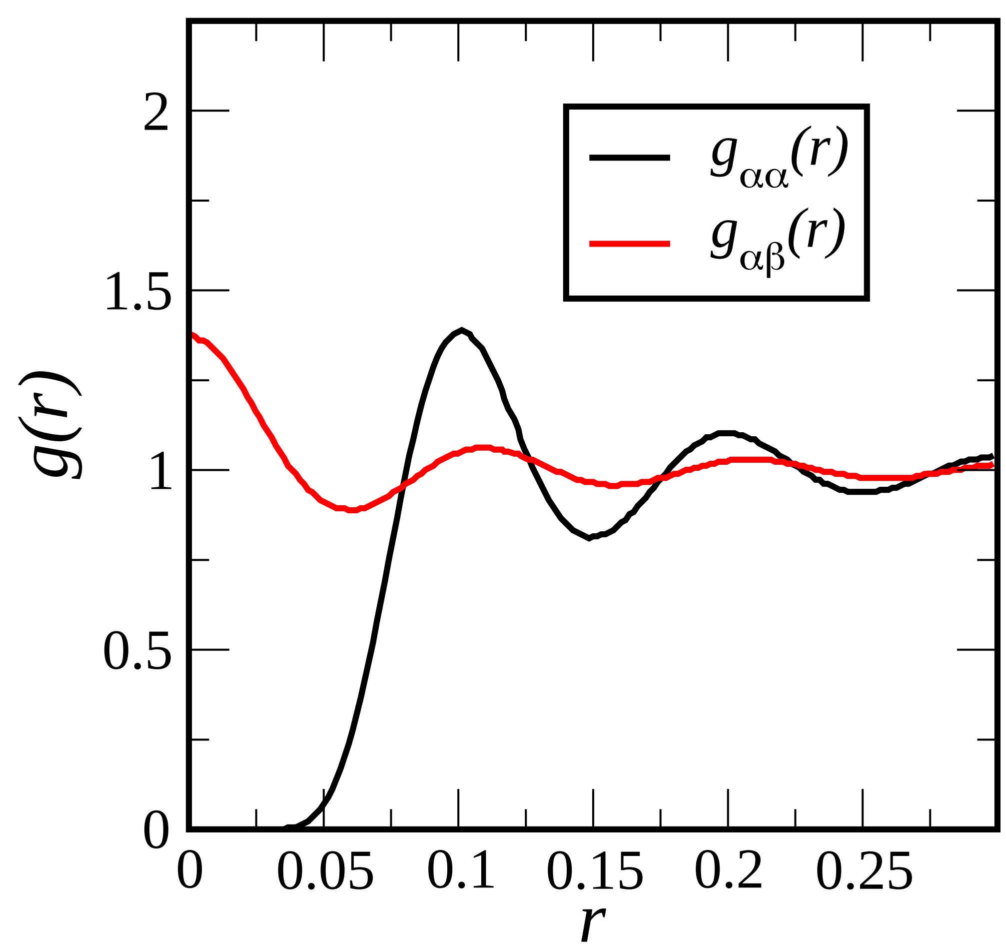


plane 1 (α)

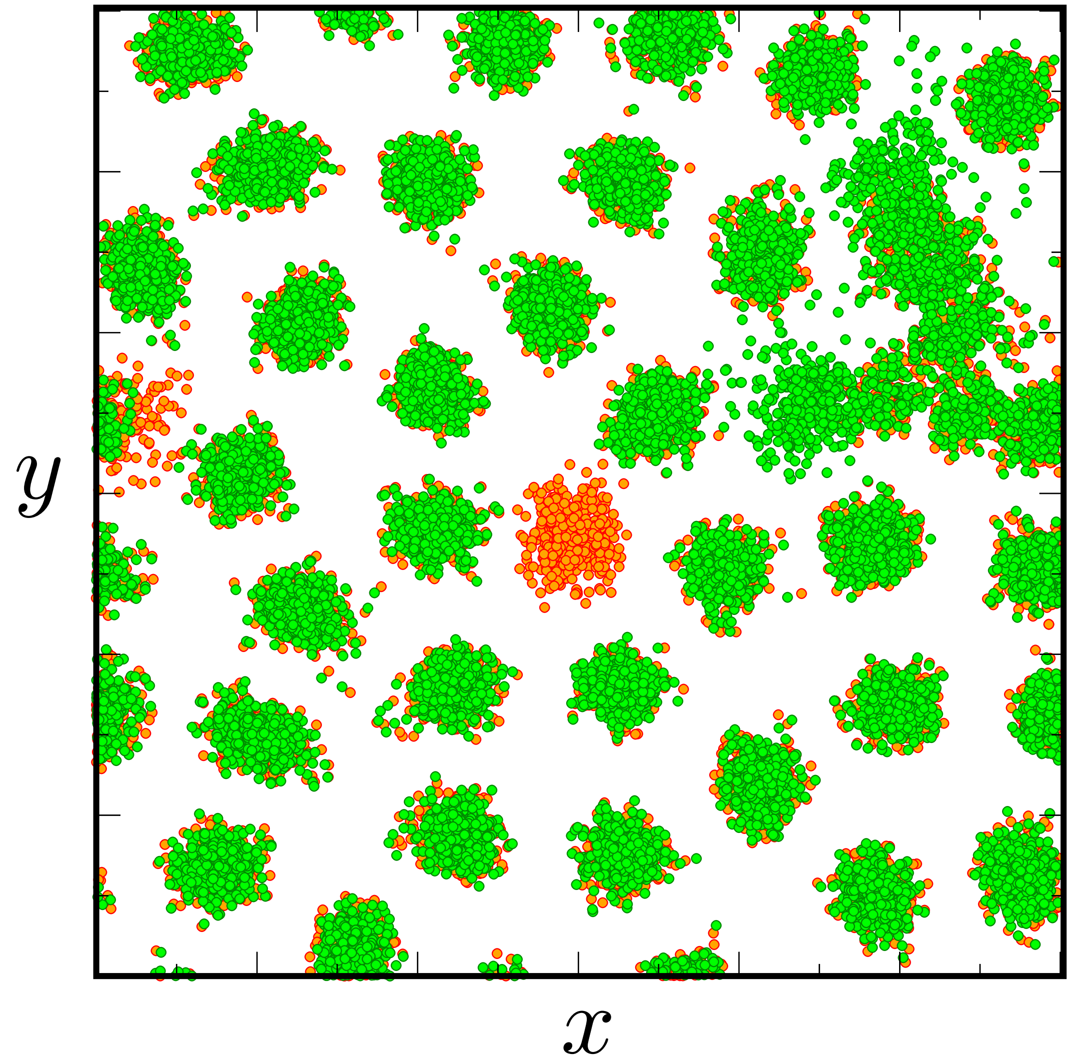
plane 2 (β)



Independent superfluids

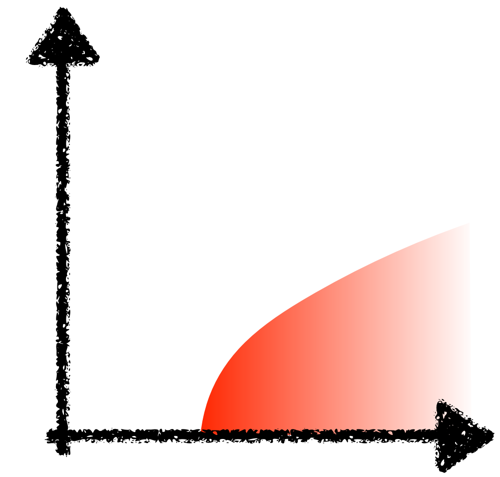


- Low-Density & planes far apart
- f_s=1.0 on each plane
- $g_{\alpha\alpha}(r)$ & $g_{\alpha\beta}(r)$

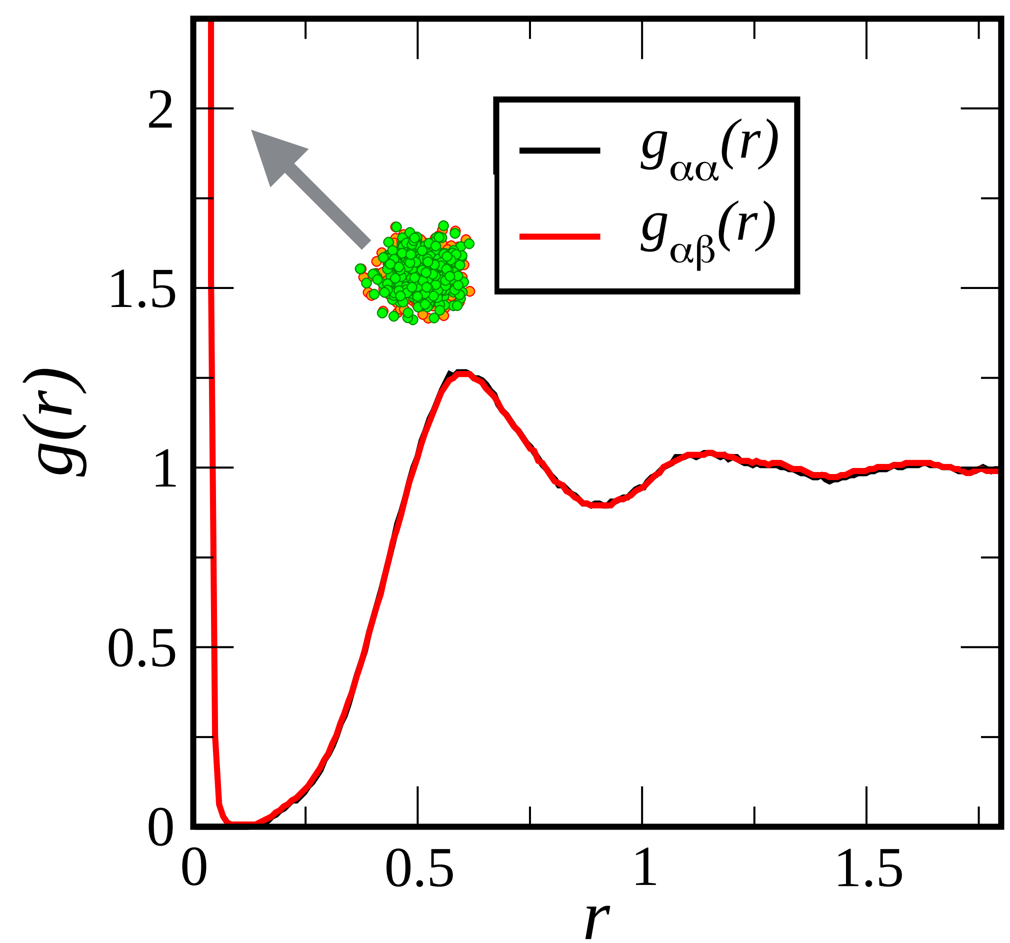


plane 1 (α)

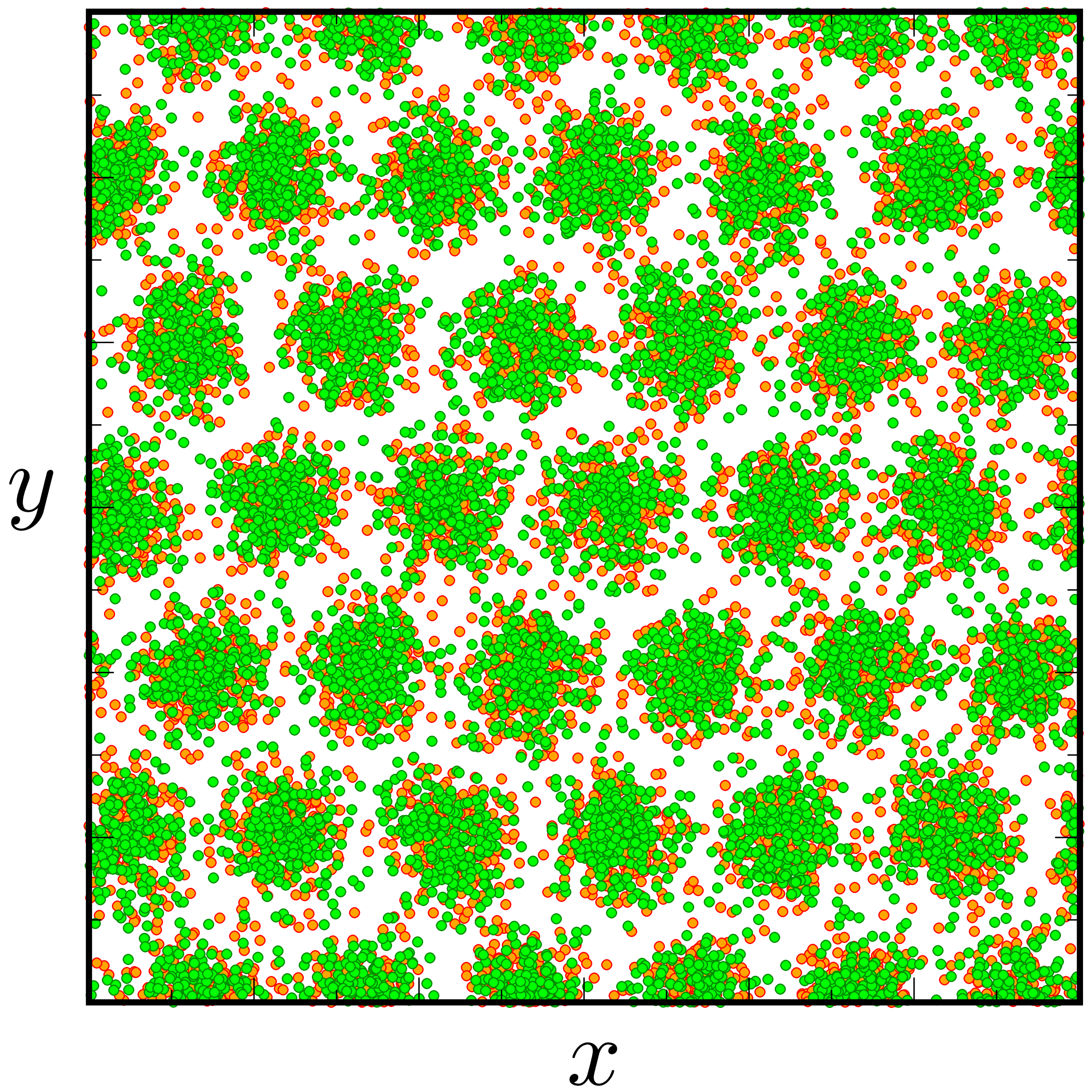
plane 2 (β)



Paired superfluid

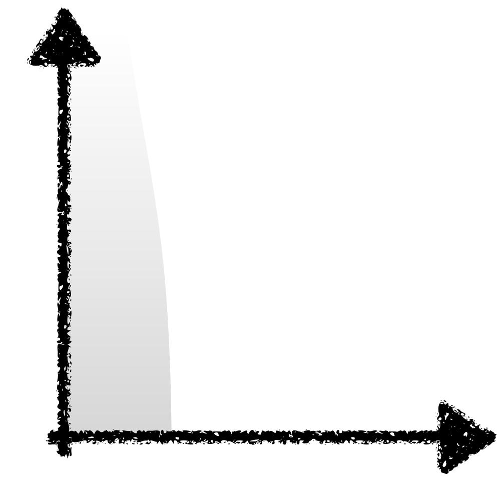


- Low-Density & planes “closer”
- f_s=0.5 on each plane
- 2D universality class (BKT transition)
- Additional strong range correlation

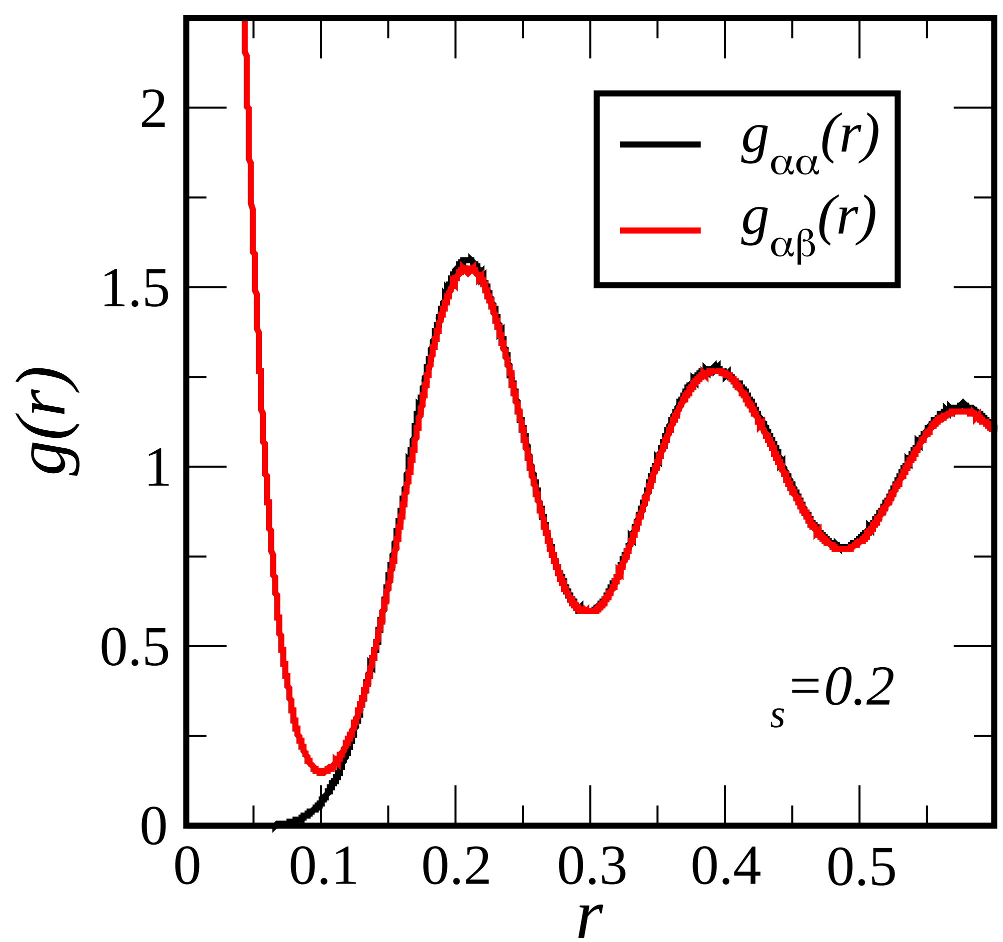


plane 1 (α)

plane 2 (β)



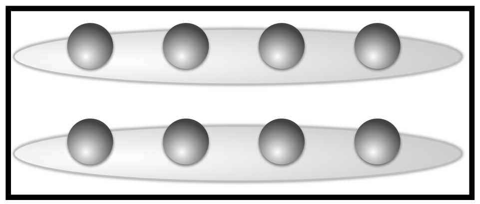
Paired crystal



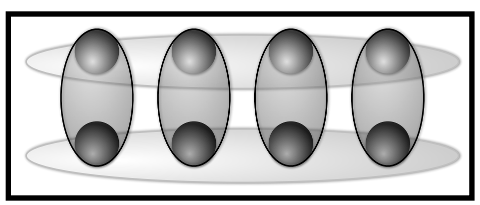
- High-Density & planes closer and closer
- f_s=0.0 on each plane

To conclude…

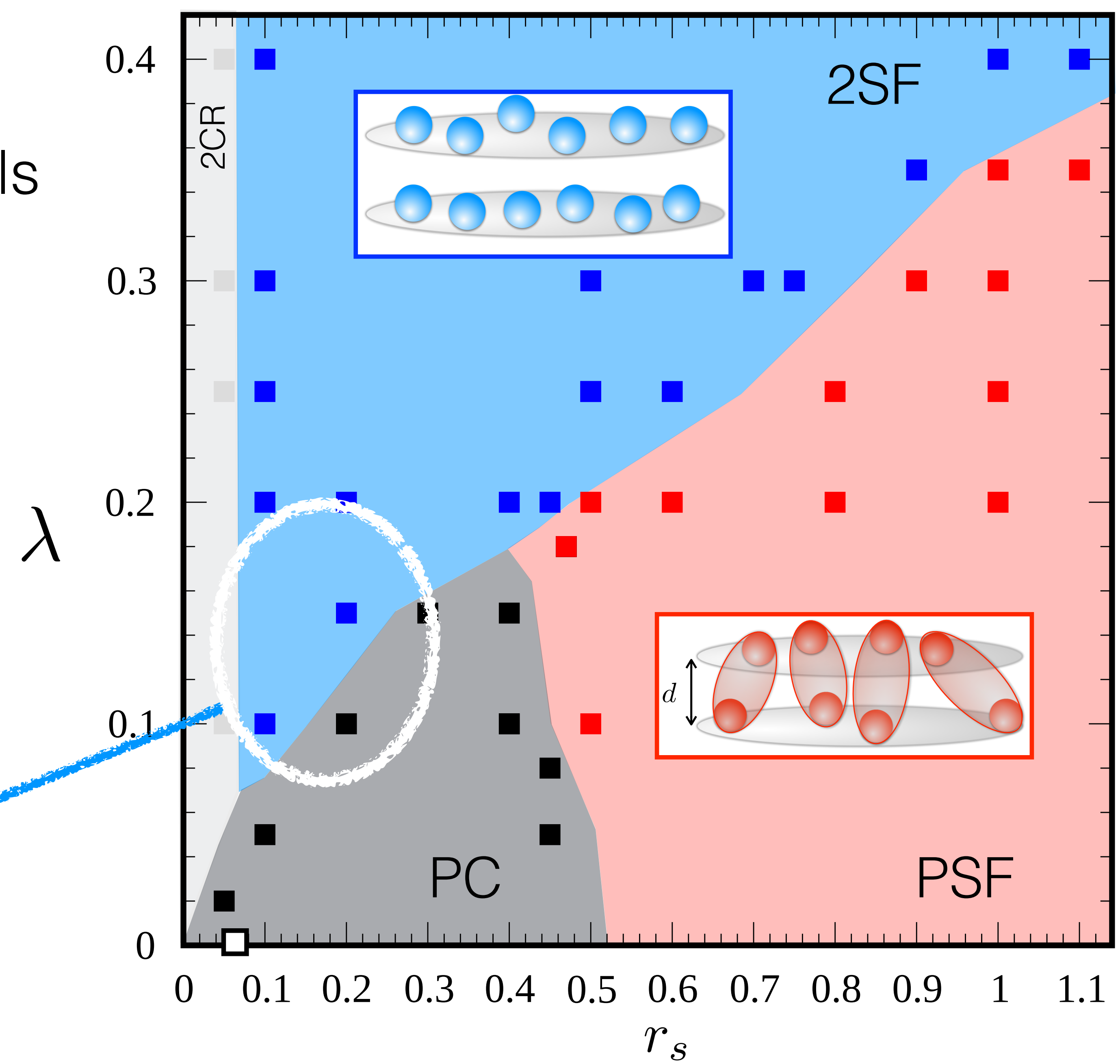
Schematic ground state phase diagram of bosonic dipolar gases on a bilayer geometry



2CR “melts” in decoupled crystals

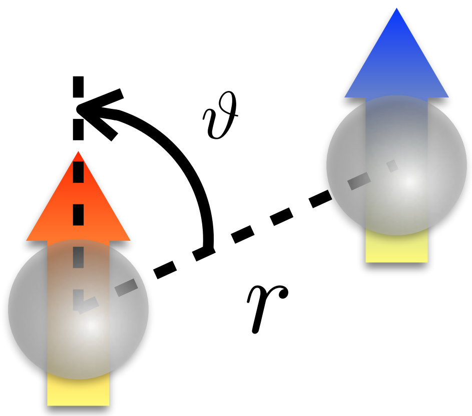


PC melts in a paired fluid

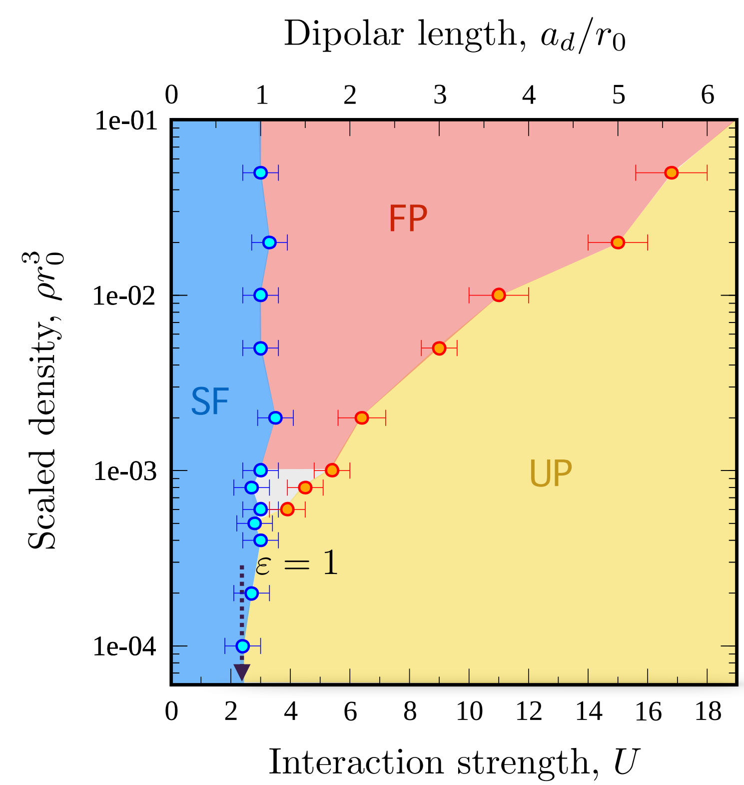


2SF extends its domain to lower λ: competition between in- and inter-plane interactions

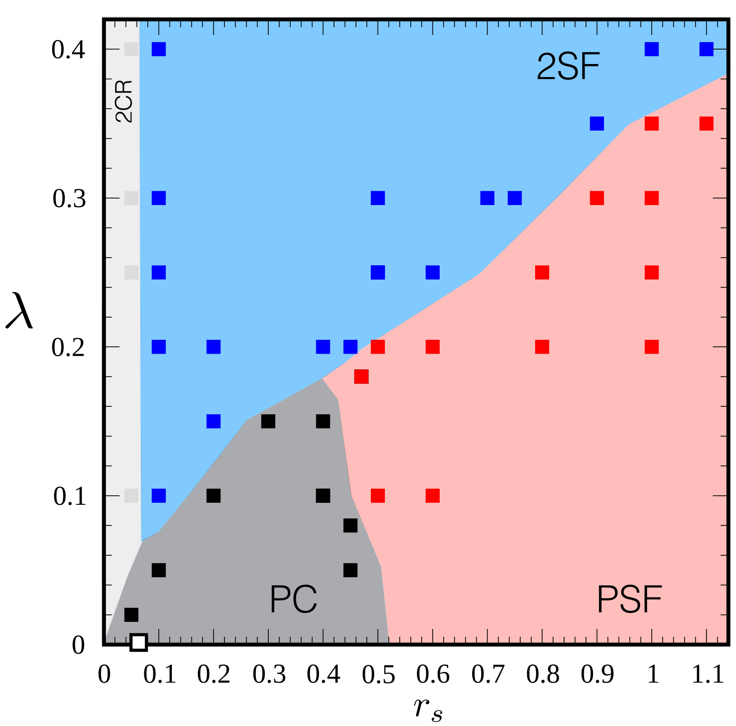
Conclusions



Stabilisation of quantum dipolar droplets is becoming an exciting research topic!



- Many-body phases of an ensemble of bosons interacting via dipole-dipole interactions
- Outlook: critical properties on trapped systems



- Quantum phase diagram of dipolar bosons on bilayers
- Outlook: feasible experiments?

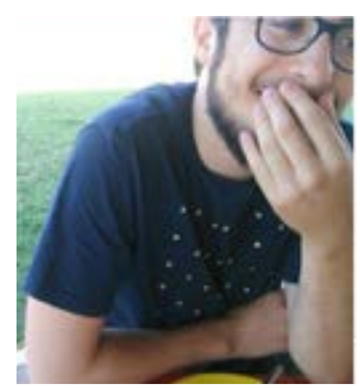
Main co-workers

Tommaso Macrì
Universidade Federal do Rio Grande do Norte (BZ)

Massimo Boninsegni
University of Alberta (CA)

Daw-Wei Wang
National Tsing-Hua University (TW)

Luca Salasnich
CNR & Università di Padova



Alberto Cappellaro
CNR & Università di Padova